Music Performance Research

Vol 7: 1-26
ISSN 7155-9219

A corpus analysis of rubato in Bach's C Major Prelude, WTC I

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ABSTRACT: We examined microtiming properties in a corpus of 48 recorded performances of J.S. Bach's C Major Prelude from The Well Tempered Clavier, Book I. Drawing on the results of a listening experiment and from wavelet analysis, we derived a quantitative measure of rubato 'depth' that was used to assess timing trends across performances. In addition to highlighting important structural moments in the Prelude, rubato was used to bring melodic elements into relief as well as to generate grouping segmentations that may contradict the Prelude's inherent phrase structure. We then applied the statistical method of principal components analysis (PCA) to examine timing contours specific to individual performers. Repetitively consistent microrhythmic patterns, which we qualified as groove-like, differed from non-consistent and non-repetitive timing inflections, which we qualified as rubato-like.

KEY WORDS: rhythm, rubato, groove, microtiming, J.S. Bach, C Major Prelude, corpus analysis

This study examines various dimensions of rubato in 48 recorded performances of J.S. Bach's C major Prelude (BWV 846) from The Well Tempered Clavier, Book I. One of a small handful of 'figural' (or 'pattern') keyboard preludes by Bach, this work is structurally unique among the keyboard compositions analysed in the literature on rubato. The Prelude's foreground lacks melodies, rhythmic motives, textural variety, and perfect cadences. Instead, for 32 of its 35 bars, the music unfolds algorithmically: a five-note ascending arpeggio, repetition of the last three notes, repetition of the last two beats, and repetition of the process with a new pitch set. "The musical teleology," writes Robert Wason, "[is] determined purely by resources of pitch organization – or 'harmony'" (2002, p. 104).

This surface homogeneity requires that we think carefully when invoking one of the widely accepted and empirically validated tenets of rubato: that it is used at phrase boundaries, often as a result of previous phrase-final lengthening (Todd, 1985). Despite the Prelude's unusual structure, its phrase boundaries are delineated harmonically, as is the case in most Western common practice music. We will therefore encounter much concordance between the Prelude's harmonic structure and uses of rubato. But there is more to the story. Our goal is to show that performers use rubato in such a way as to seem to extract linear elements from vertical structures, shift the focus of the musical narrative, lock into a groove, and (yes) delineate phrase boundaries.

We do this by considering the duration of each semiquaver in the context of those surrounding it, so as to gauge the 'depth' of the rubato. Thus, a one-dimensional value denoting rubato magnitude is assigned to each individual note, even though rubato typically consists of a timing gesture encompassing several notes. Such compression reduces the number of data points and facilitates quantitative comparisons within and across performances.

Next, we 'unpack' magnitudes of rubato to reveal their internal makeup in the form of note-duration profiles: sequences of note durations that comprise a specific timing contour. The concept of rubato is thus seen as a suite of distinct gestures, each with its own microrhythmic 'flavour.' By treating these profiles as vectors in a multidimensional space, we refine Povel's (1977) and Cook's (1987) claim that different performers time the Prelude's recurring metrical framework in different ways. Aided by a much larger corpus, a more rigorous quantitative arsenal, and a more thorough consideration of perceptual thresholds, we offer an analysis that posits rubato as a complex family of interacting temporal behaviours.

As we examine these temporal behaviours closely, we will ultimately argue in favour of making a distinction between two microtiming features that, while conceptually distinct, may coexist within a given performance: rubato and groove.[1] Whereas rubato consists of a momentary temporal dilation or compression with respect to the prevailing tempo, a special case of this effect is that of a groove: a microrhythmic pattern that cycles repeatedly following the same temporal profile each time.[2] The term 'groove' is typically encountered in discussions of dance-oriented music (Butler, 2006; Feld, 1988). By applying the term in an atypical context (Baroque keyboard music), we hope to draw a connection between two temporal effects that are not usually studied in conjunction.

Broadly speaking, the present study addresses two lines of inquiry related to the performance of rubato. The first line of inquiry concerns how classical pianists use timing to highlight different structural features of a given composition. Recent work on rubato, especially Ohriner (2012), sheds light on the multiplicity of possible structural readings afforded by a composition's underlying formal structure. This stands in contrast to the view that timing fluctuations serve to communicate *the* underlying formal structure of a composition (e.g., Todd, 1985). The study's second line of inquiry concerns performer

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[1] Penel and Drake (1998) have shown that it is possible for two distinct microtemporal properties to coexist within a musical performance. The authors propose that higher- and lower-level processes influence the music's hierarchical and local timing patterns, respectively.

[2] This distinction is different from the two types of rubato discussed by Hudson (1994), where 'Type 1' rubato pits a temporally variable (usually melodic) layer against an isochronous (usually accompanimental) one, and 'Type 2' rubato affects the entire musical surface.

identity as a product of individual timing choices (e.g., Koren & Gingras, 2014). Repp (1992a, 1998b) lay the groundwork for the statistical analysis of rubato in large corpora consisting of recordings by multiple pianists. Here we extend that work by including additional types of keyboard instruments (harpsichord and clavichord) and – more importantly – by considering a musical texture that features extensive pattern-based repetition.

QUANTIFICATION AND PERCEPTION

We begin with an overview of the timing data in our corpus: note-onset times in 48 different Prelude recordings by highly accomplished musicians (listed in Appendix A). These recordings span a period of 80 years and therefore provide a comprehensive sample – though by no means an exhaustive one – of the plausible uses of rubato in the Prelude. Nineteen of the recordings were made on harpsichord or clavichord; the rest on piano. Since automated means of onset detection yielded inconsistent results, note onsets were marked manually by the first author using what are nowadays fairly standard techniques in microtiming analysis: aided by waveform and spectrogram displays, each onset is localised as accurately as possible by identifying the beginning of the note's onset ramp. Measurement errors are estimated to lie within ±5 ms, although in some instances this figure may be doubled or (seldom) tripled, particularly in the case of low-fidelity recordings from the 1920s and 30s. These measurement errors can be considered negligible in relation to the temporal effects under study.

No performance in the corpus is free of temporal oscillation of some kind. Therefore an initial step in our analysis consisted of filtering out imperceptible irregularities in the timing data. A straightforward automated way to assess whether a given moment of music qualifies as rubato is to set a minimum threshold for duration differences between adjacent elements. The main drawback of this method is that it can be 'fooled' by very smooth tempo changes: gradually decelerating (or accelerating) notes will be similar enough in duration to sneak by undetected. A related technique to do away with low-level fluctuations is to calculate moving averages and look for conspicuous spikes. This approach proved fallible as well, as the window size of the moving average was either too narrow to detect relatively spread-out decelerations or too wide to detect highly localised ones. We are reminded of Bengtsson and Gabrielsson's (1983) characterisation of tempo as a multi-level perceptual construct. Tempo permeates note, beat, bar, section, and piece. Constructing a model that faithfully reflects the perceptual complexities of this temporal conglomerate is beyond the scope of this study (even if we restrict ourselves to the more local levels only).

As a promising alternative, we implemented a discrete variant of the Ricker wavelet, also known as the 'Mexican hat' function for reasons made evident in Figure 1(a). Mexican-hat averages provide a standard technique for the detection of statistically significant peaks in otherwise noisy time series. They are extensively used, for instance, in the analysis of seismic data (Ricker, 1953) and temporal signals of biological origin, such as neural activity (Zainuddin, Huong, & Pauline, 2012). Our method assigns each note in the Prelude a rubato index R – essentially a measurement of rubato magnitude that can be interpreted as a relative lengthening with respect to the local context.[3] The value of R at the *n*th note is a

[3] Even though R is responsive to note lengthenings and shortenings, this study focuses on lengthenings

weighted average of the interonset intervals (henceforth 'durations') of 24 consecutive notes, from the (*n*–11)th to the (*n*+12)th. For instance, for the last note of each bar, the average includes the last 12 notes of that bar and the first 12 notes of the following one. (See Appendix B for more details.) Once R was evaluated for each note in every performance, it was compared to a threshold t determined from a perception experiment, as described below.[4] This allowed us to focus on the perceptual aspects of the timing fluctuations.

Figure 1(a) depicts the weight of each note duration; the average ascribes positive weights to the four central notes (*n*–1 to *n*+2) and negative weights to the others. Figure 1(b) shows that R (as set by the Mexican hat function) is sensitive to context, so that any two notes with the same duration do not necessarily yield the same value of R. The unfilled dots in the graph correspond to note durations in an excerpt from Till Fellner's performance; the filled dots denote R values. Even though the two notes marked x and y have identical durations (220 ms), note y has a much higher R value than note x (that is, it has greater rubato depth) as a result of y's proximity to longer notes.

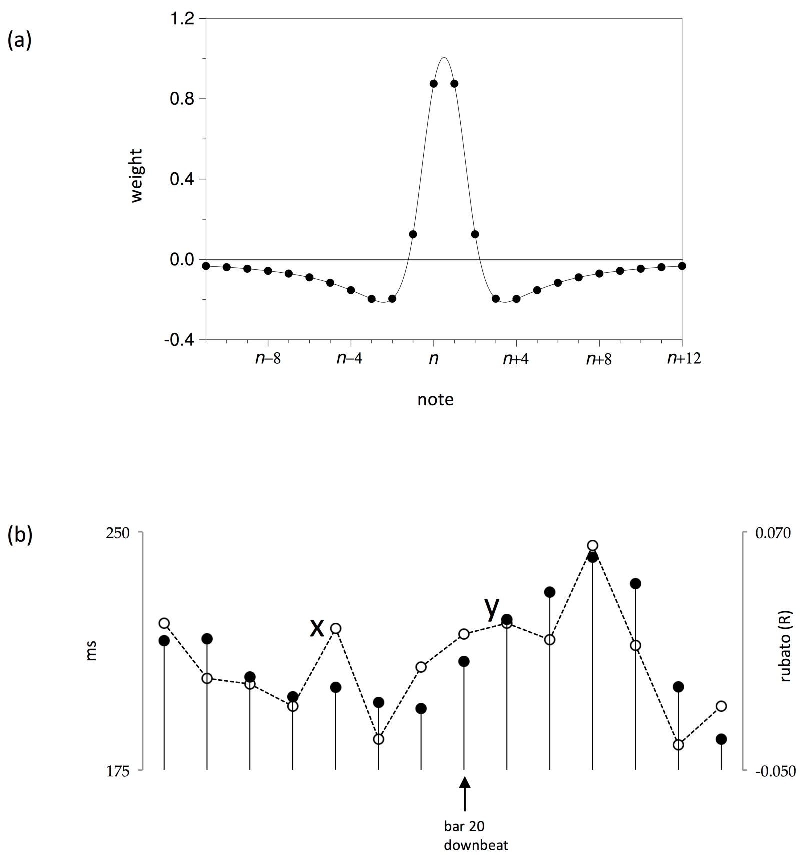

Figure 1. (a) A variant of the 'Mexican hat' function, with weight values assigned to each semiquaver within a moving window of 24 notes. (b) Conversion from durations (unfilled dots) to R values (filled dots) in a 14-note excerpt from Till Fellner's performance.

exclusively because Prelude performances almost never accelerate locally.

[4] All measurement units (note durations, R, and t) are in seconds. R is biased to yield higher values for slower performances, since the calculation takes as input absolute duration values.

If rubato is a perceivable tempo alteration, then we need a way to filter appreciable rhythmic inflection from unperceivable noise in the timing data. Performing a computational analysis of rubato entails determining which temporal irregularities are thus subsumed. This remains an open challenge that has been summarily bypassed by almost all quantitative analyses of rubato.[5] These studies affirm the temporal flexibility of performers while ignoring – or making assumptions about – the cognitive capacity of listeners. A few researchers have danced around this question without tackling it head-on. Johnson (2003) presented musically trained subjects with six alterations of a recorded Bach excerpt. Subjects rated the original to be "more musical" than the versions with higher or lower amounts of rubato. This obviously implies that listeners could differentiate (perhaps subconsciously) between degrees of rubato in different performances, a claim that is of limited use for our present purposes. A related conclusion was reached by Honing (2006), who asked musically trained participants to listen for expressive timing when distinguishing tempo-transformed versions of classical piano performances from the original recordings. Correct identification was at above-chance levels; it remains a mystery what specific timing cues listeners relied on for their judgments.[6] Finally, Repp (1992b, 1998a) showed that changes in note duration are more readily detected when the affected note lies away from a structural boundary – that is, where a listener would least expect a performer to lengthen or shorten the note. While Repp used excerpts by Chopin, Beethoven, and Schumann, these were metronomic MIDI renditions that lacked the temporal variability found in real performances, even those played without rubato.[7]

In short, we were unable to identify a behavioural study where listeners report the presence of rubato along the performance of a given musical work.[8] We therefore conducted a listening experiment to assess the extent to which our rubato index R reliably reflects the perception of rubato in the Prelude.

METHOD

Participants

Four professional musicians participated in the experiment: two pianists, an orchestral conductor, and a composer (the first author); mean age = 35 years, range = 32-39. All participants hold advanced music degrees and are actively involved in the study and performance of Western classical music. They were paid for their participation (except for the author).

[5] Some examples include Cheng and Chew (2008), Clarke (1985, 1995), Cook (1987), Palmer (1996), Povel (1977), Repp (1992a), Shaffer (1992), Timmers et al. (2000), Todd (1985), and Widmer (2002). Povel closes his study of microtiming in the Prelude with a promise of follow-up experiments that would "determine the perceptual relevance of the different characteristics of the temporal structure" (1977, p. 318). To our knowledge, the results of those experiments have not been published.

[6] In some instances, listeners may have detected timbral artifacts resulting from the transformation of the audio file; see Honing's Experiment 2.

[7] There is a substantial amount of research on the perception of timing perturbations in non- and quasi-musical passages. See Drake and Botte (1993), Friberg and Sundberg (1995), and Repp (1995, 1998a Experiments 4 and 5).

[8] Perhaps Butterfield (2010) comes closest, although rubato was not the topic: listeners gauged asynchronies between bass and drums in jazz performances.

Stimuli

Twelve recordings of the Prelude were randomly selected from the full corpus of 48. Each participant received a take-home audio CD containing these selected performances (marked with asterisks in Appendix A). No information was given about the identity of the performers; tracks were labelled Track 1, Track 2, etc. Participants also received a binder with copies of the musical score, along with a set of instructions.

Procedure

For each recording, participants marked instances of perceived rubato on the corresponding musical score. Rubato was defined as a "perturbation of metronomic sixteenth-notes [i.e., semiquavers] that may take different forms: as a ritardando, as a single note held for longer than written, etc." Participants were instructed to focus on the region near the barlines (the reason for this will become clear below). Each barline received a rubato rating as follows:

> 0 No rubato at all. This score was used when there was no discernible expressive deviation in the rhythm – that is, in instances of isochronous semiquavers.
>
> 1 Almost imperceptible rubato. Participants used this score when unsure as to whether any rubato was present, or in cases where rubato was so subtle as to require repeated listenings to catch it.
>
> 2 Clearly audible rubato. This score applied to all manners of salient rubato, including varying amounts and types. A very mild and an overtly exaggerated rubato would both receive a score of 2.

This was done up to the downbeat of the 28th bar (thus excluding the last five bars) as a way to keep the task more manageable for the participants and to exclude the final ritardando. Participants were encouraged to listen to each recording as many times as necessary in any order they wished, and to devote careful attention to the scoring process. They were also instructed to ignore articulation and dynamics. On average, participants took approximately two hours to complete the task.

RESULTS

We compared the perceptually-derived ratings of rubato with the quantitatively-derived rubato scores (as set by R). This enabled us to determine a threshold value t that would maximize R's correlation with the participants' averages. Correlations were calculated between 324 pairs of perceptual ratings and R values: 27 barline rubatos x 12 recordings.[9] The strongest correlations were found when $t = .045$ ($r = .79$, $p < .00001$). The correlations between individual R values and mean participant scores were high ($r > .75$) for ten of the 12 versions – Jarrett ($r = .5$) and Pollini ($r = .3$) being the two exceptions. With R's threshold t thus set to .045, we proceeded to analyse the entire corpus of recordings.

[9] In the calculation of the correlation, as elsewhere in this study, we set R values to zero when $R < t$ and kept R values intact when $R \geq t$.

THE ROLES OF RUBATO

There is general consensus among music theorists concerning the Prelude's formal structure (Cook, 1987; Drabkin, 1985; Lerdahl & Jackendoff, 1983; Lester, 1998; Schenker, 1933/1969; Wason, 2002). Figure 2 gives a structural sketch adapted from Lester (1998); the full score forms Appendix C. There are two main sections between the outer cadential frames. The first section is a harmonised bass line that descends from middle C one octave by step. It is further subdivided into two eight-bar halves, the second (bars 12 to 19) being a transposed elaboration of the first (bars 4 to 11). Next is a four-bar bridge leading to the second main section, an eight-bar dominant pedal that again splits into two halves. The elision at bar 8 is the only outlier in the otherwise regular flow of four-bar hypermeasures.[10]

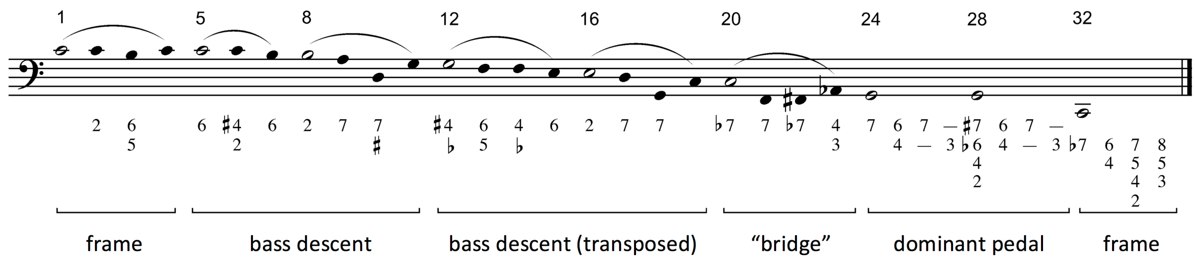

Figure 2. Structural reduction. Following an introductory pedal point on middle C, the bass line descends one octave by step (bars 5 to 20). This leads to an eight-bar dominant pedal (24 to 31) followed by a tonic pedal (32 to 35) that recalls the opening four bars.

Since the majority of rubatos occur around barlines, we assign each barline the value of R corresponding to the last note of the preceding bar and (for now) discard all others.[11] Figure 3 shows R values for the last note of each bar averaged across all performances (henceforth 'average rubato profile'), starting with the barline at 1-2 and ending with 31-32, right before the arpeggio pattern breaks down at bar 33. (Recall that R values below $t = .045$ are converted to zero; this explains the relatively low averages shown in the graph.) The highest peaks at 23-24 and 31-32 correspond to the 'deepest' grouping boundaries in Lerdahl and Jackendoff's time-span reduction of the Prelude (1983, p. 262) – namely, the respective arrivals of the dominant and tonic pedals. The four-bar hypermeasures beginning at bars 8, 12, 16, and 20 are borne out by secondary peaks in the average rubato profile. Further segmentation reveals that these peaks are joined by tertiary peaks at 9-10, 13-14, and 17-18, tracing a see-saw pattern that conforms to the two-bar harmonic pattern within the hypermeasures.

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[10] This is where Lerdahl and Jackendoff (1983) place the elision, whereas Schenker (1933/1969) and Wason (2002) prefer bar 4. The decision concerns whether to group the opening bars as 3+4 (elision on the fourth) or 4+3 (elision on the eighth). We also note that Komar's (1971) analysis gives hypermetric downbeats at bars 7, 11, 15, 19, and 23 – all one bar early compared to the analyses mentioned above.

[11] Evaluation of R for all notes in all recordings up to the downbeat of bar 32 returned a total of 1186 notes where $R \geq t$. From those, more than 60% corresponded to either the first or the last note of a bar, and almost 94% belonged to either the first or the last beat of a bar. These two groups constituted, respectively, 65% and 97% of the total amount of R across all performances. It is therefore clear that most instances of rubato took place in the vicinity of a barline.

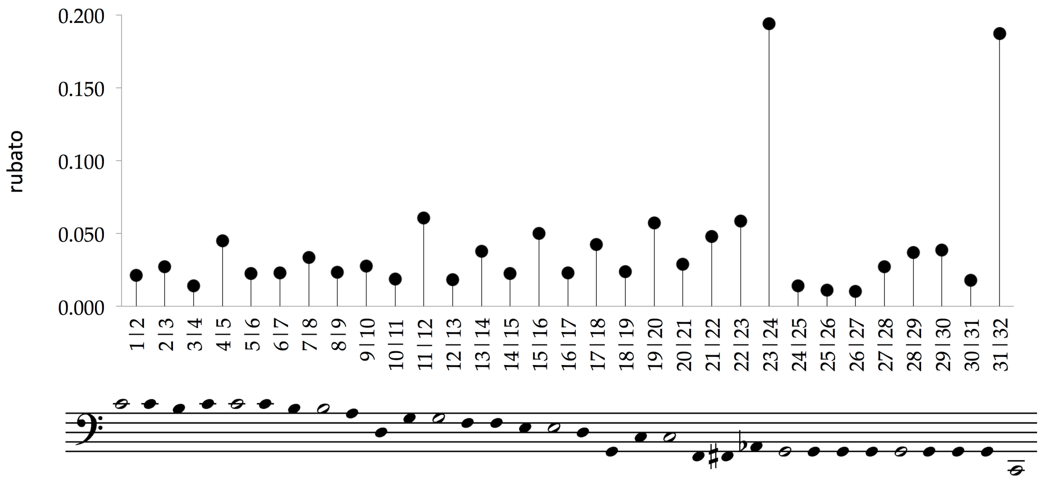

Figure 3. R values for the last note of each bar averaged across all performances. There is greatest rubato at barlines that are structurally important.

Analogous correlations between rubato profiles and harmonic structure have been amply addressed by Clarke (1995), Repp (1998a, 1998b), Todd (1985), and others. Of course, each performer employs rubato in his or her own way, and many performers deviate considerably from the average rubato profile. In fact, there is not a single performance that perfectly articulates the hypermetrical breakdown seen in the average rubato profile. (Nor does anyone blatantly contradict the structural hierarchy; there are fewer unlikely profiles that came to pass than likely ones that did.) This led Cook (1987, p. 266) to note a "not particularly good" correlation between the Prelude's structure and the seven performances he analysed.[12] Figure 4 gives several contrasting cases from our corpus. Riefling closes off the opening frame with a clear caesura at 4-5. Fellner highlights only the two major structural boundaries (23-24 and 31-32). Hewitt uses a lot of rubato at 13-14, a structurally unimportant location. Jarrett favours the arrival of the flattened submediant (at 22-23) over that of the more popular dominant in the following bar. Gould, like others, dispenses with rubato entirely until the tonic pedal at 31-32. Fischer and Gieseking (not shown) play isochronously throughout, whereas Egarr adds rubato everywhere except at 29-30 – precisely the barline at which Feinberg uses most rubato.

What are we to make of those rubatos that coincide with structurally unimportant locations? To dismiss them as inconsequential or idiosyncratic seems hardly fair. The performer's attention (in the form of rubato) to apparently minor details of the score surely deserves commensurate attention from the analyst. Hewitt's solitary and pronounced rubato at 13-14, for instance, suggests a segmentation that conflicts with that of the underlying structure.

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[12] Cook (1987) classifies every bar as either having or not having "phrase-final lengthening;" *cf.* Fig. 6c (p. 265). He offers neither data nor an explanation of how he derived his binary system of classification.

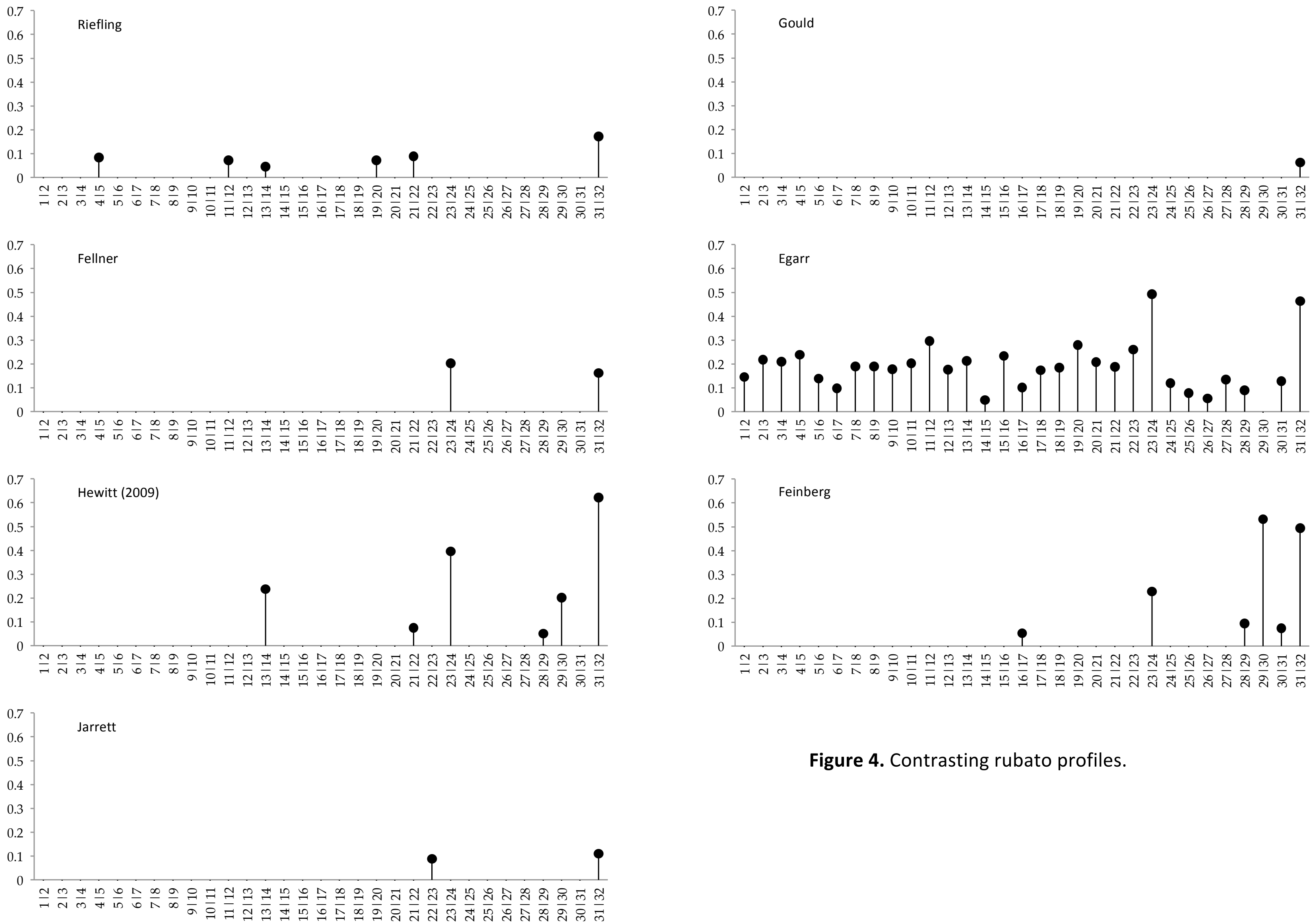

Figure 4. Contrasting rubato profiles.

A number of writers have argued that the pacing of a work can be regulated by different approaches to timing (e.g., Clarke, 1995; Ohriner, 2012; Shaffer, 1992). This mode of analysis, in which timing cues serve to define particular readings of a composition's structure, has mostly displaced the earlier belief that rubato merely communicates a pre-existing formal hierarchy. Cook (1987, p. 267) also refutes such 'over-determinism' on the grounds that it is based on the model of an ideal performance which any actual performance will approximate to a greater or lesser degree, whereas what would seem more appropriate would be to correlate hierarchical structure with some kind of durational framework within which different surface realizations would be possible.

In our view, a non-aligned rubato can cause the kind of perceptual discord akin to the grouping dissonance brought about by non-aligned metrical layers (Krebs, 1999). For such rubato-based dissonance to occur, one needs both a firm conception of the work's abstract structure as well as a flesh-and-blood agent who defies that conception. Hewitt's non-aligned rubato at bars 13-14 creates the effect of making a boundary that might otherwise be heard as superficial sound more important in terms of the work's structure. The performer is not only defining a structural reading; she is also juxtaposing that reading in manifest opposition to the work's immanent form.

While rubato may encourage the listener to segment, we do not maintain that all non-aligned rubatos automatically do so. Some may bring to the foreground melodic features that are safely contained within the structural group. A hypermeasure may be inflected microrhythmically without altering its underlying structure.[13]

Figure 5(a) shows that Verlet's (mild) rubatos at 2-3 and 6-7 mark the slipping downwards of the bass line from the tonic, calling attention to the imminent octave descent. The rubato appears to be driven by the linear motion and poses no threat to the integrity of the grouping. Instead, the effect is one of peeling a linear strand off the chordal surface.[14] Note that Verlet plays without rubato at phrase boundaries nearby – where rubato might be expected – in order to bring the linear effect into relief. Backhaus's non-aligned rubatos at 8-9 and 16-17 also seem melodic in nature (Figure 5(b)). The supertonic-to-tonic resolution of the bass suspension deems the *root* of each ii7 (not the entire chord) as an arrival worthy of noting. The same can be said of his rubato at 20-21, where again our attention is drawn to the arrival of the root – this time via leaping – instead of the F major chord, which only materialises partially, in any case. Backhaus thus brings to the foreground the bass's melodic strand by picking out specific linear resolutions and bypassing the hypermetric landmarks.

Several performers in our corpus use rubato to emphasise the melody in the soprano voice over the eight bars of dominant pedal. The highest voice traces a stepwise ascent from D4 (bar 24) to G4 (bar 29), then reverses direction to reach a quasi-resolution at E4 on the tonic (bar 32). Because the top line must always wait its turn until after the others have each contributed a semiquaver to the arpeggio, the following analysis ventures beyond the barline and toward the second beat of the bar.

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[13] See Cone (1968, p. 64) for a linear treatment of the Prelude's harmonically ambiguous 23rd bar.
[14] Yeston (1975) offers a brief discussion of how harmonic rhythm can be defaced by extreme rubato.

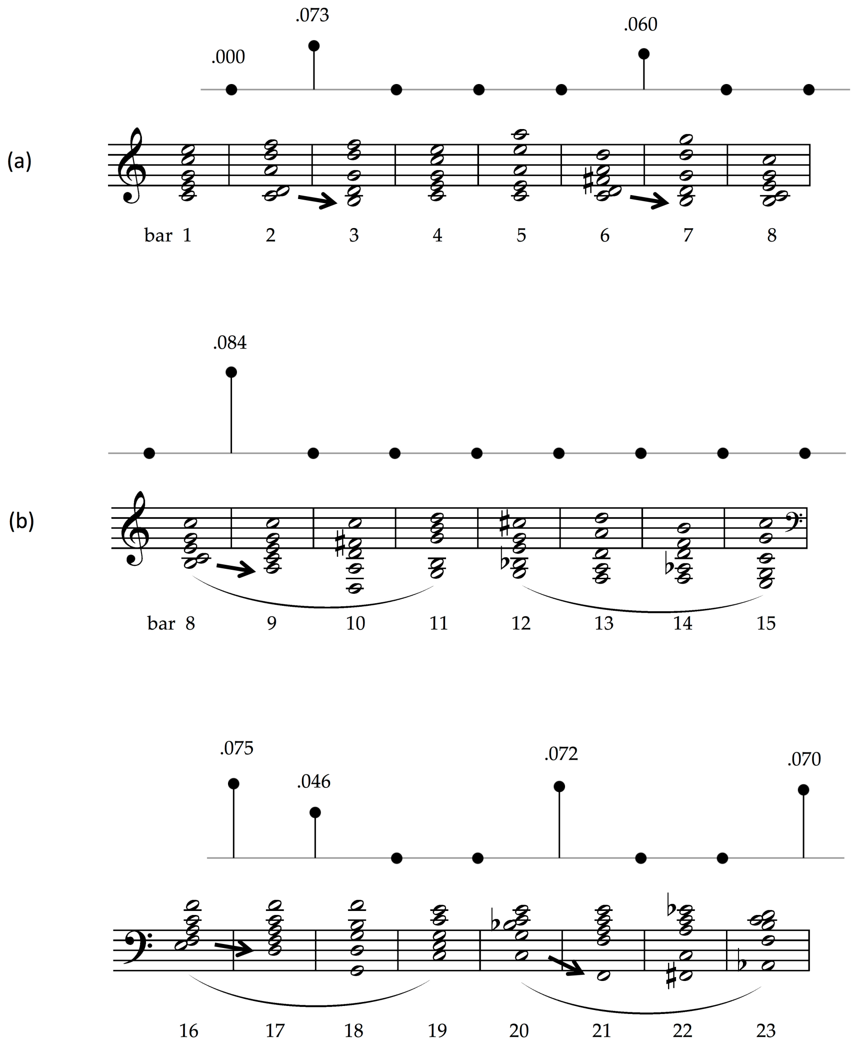

Figure 5. Line-based rubatos at non-boundary locations. (a) Verlet emphasises the linear bass motion from C to B. (b) Backhaus highlights two suspension resolutions (B to A, E to D) and a descending leap (C to F).

Figure 6 shows mean R values for the Prelude's fifth semiquaver (the onset of beat 2) over the dominant pedal. The bracketed numbers indicate how many performers use rubato at that location. As the line ascends, there is a build-up of rubato that culminates in a striking peak at the melodic summit on G4. If performers' rubatos were primarily concerned with communicating the Prelude's grouping structure, then we would expect there to be less rubato at bar 29 than at bar 28 (a hypermetric downbeat) or bar 30 (the binary division of the hypermeasure).[15] Conceiving the melodic summit as a hard-earned goal is consistent with the shift in rubato from the barline to the second beat. More broadly, this suggests a corresponding shift in the musical narrative from the primarily harmonic heuristic of the earlier sections to a primarily melodic one.

[15] Bar 29 is a second-inversion tonic triad but it does not function as a cadential six-four, which precludes it from gaining structural prominence.

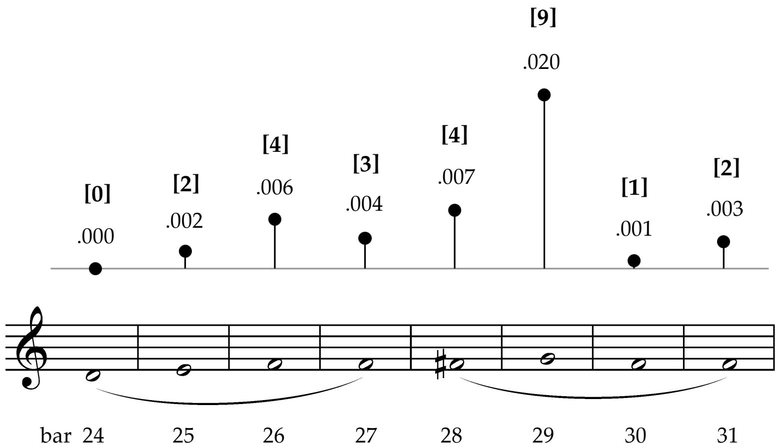

Figure 6. Mean R values for beat 2. Values are highest at the melodic peak. Bracketed numbers give the number of performances containing rubato at those locations.

As mentioned earlier, there is considerable variety between the rubato profiles of different performances. These differences can be illustrated with 'birds-eye' representations such as those shown in Figure 7. Each narrow strip corresponds to a bar, from 2 through 31, inside which 16 subdivisions are shown; the half-bar, or onset of beat 3, is marked with a coordinate line that bisects the graph. The height of the terrain denotes the amount of rubato as determined by each semiquaver's R value (if ≥ t). The graph of Levin's performance shows that, unlike the other performers, he uses rubato on the downbeats (immediately after the barline), rather than around the barline. Elsewhere in his graph, the flat landscape results from Levin's unvarying tempo within the bar. The dominant pedal (marked with a dotted line) shows a clear displacement of rubato from the first to the second beat at the start of the melodic ascent. Levin's performance thus features a striking shift from rubato on the downbeats to the second beats.[16]

By contrast, rubato in Gilbert's graph appears to straddle the barline, with peaks marking both the beginning and the end of almost every bar. As in Levin's graph, the shift to the second beat happens at the dominant pedal. However, Gilbert uses rubato to emphasise an earlier segment of the melodic ascent (bars 25 and 26), after which he reverts to using rubato around the barline. Finally, the double 'hump' at bar 29 of Tilney's graph results from his use of rubato not only on beat 2 but also one beat later. Note that Tilney's graph contains additional (small) peaks that are not around the barline; these correspond to the high G5 (a melodic resolution to the local tonic) at bar 7, the mid-bar D-minor at bar 13, and the famously late-resolving E suspension at bar 21.

Hence, while rubato at the barline is prevalent throughout the Prelude, rubato around beat 2 is reserved primarily for the ascending melody above the dominant pedal. This reinforces the idea that rubato can be prompted by harmonic as well as melodic elements. The distinction is of course noted in rubato analyses of homophonic passages, where prominent melody notes are lengthened (or shortened) in the absence of harmonic change.

[16] The visualisation employs continuous smoothing based on a discrete lattice of 16 (notes) by 30 (bars).

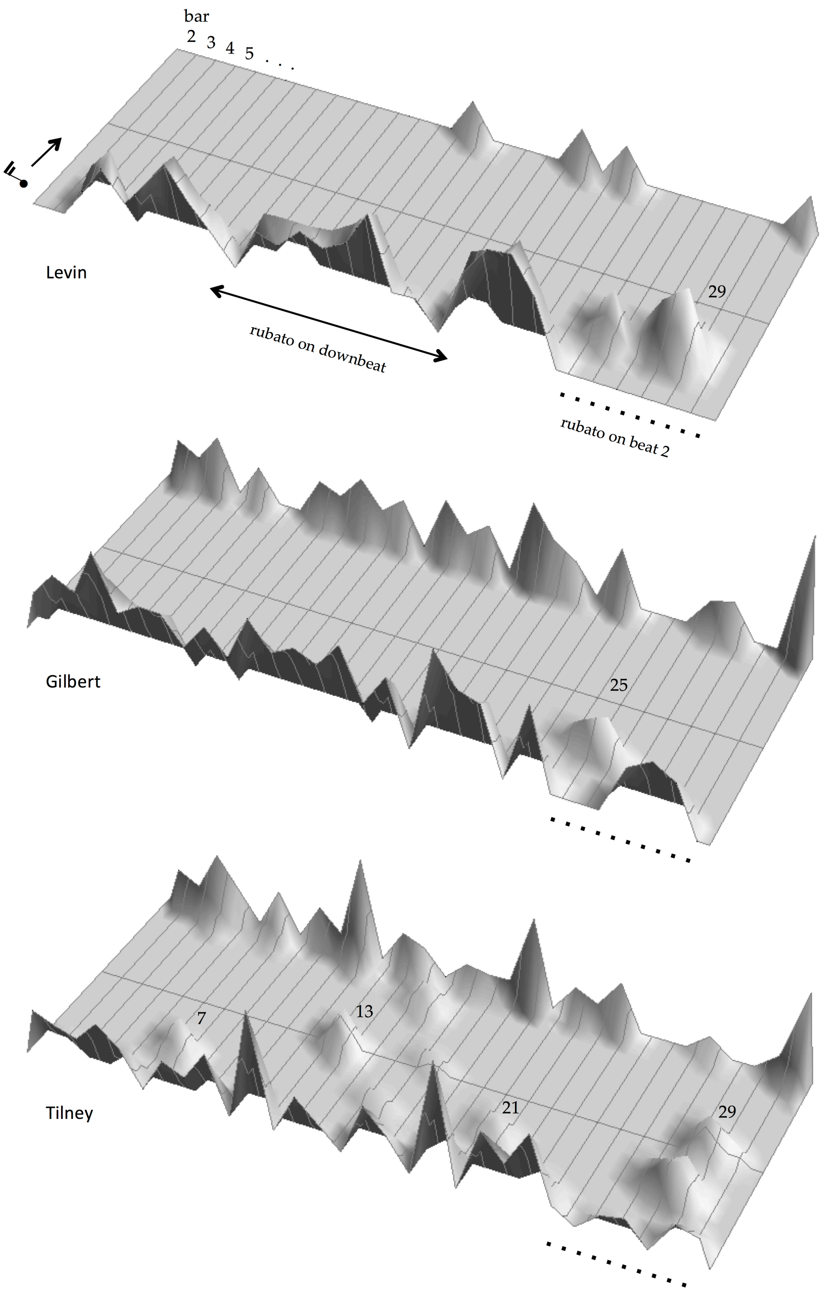

Figure 7. Aerial view of rubato, bars 2 to 31. At the dominant pedal (dotted line), rubato shifts from the barline region (or the downbeat, in Levin's case) to beat 2.

But in those cases, it seems hardly surprising that melody and accompaniment would occasionally undergo independent microtemporal tweaking, since the two elements are clearly differentiated at the textural surface. The Prelude is different, in the sense that it combines melody and harmony in such a way that they are almost indistinguishable from each other, allowing performers to use rubato to emphasise one or the other at different times within the piece. This is one reason why the Prelude provides fertile ground for microtiming analysis: the uniformity of its design amounts to what might be thought of as a set of carefully controlled experimental conditions. Cook (1987) asserts that the Prelude "makes it possible to study the effect of higher-level structure upon rhythmic nuance without getting bogged down in the complexities of how performers project foreground contrasts of rhythm" (p. 257). That this is so should not distract us from the fact that rhythmic nuance can be something other than an effect of higher-level structure or foreground contrasts.

RUBATO CONTOURS

Earlier, we defined R as the weighted average of 24 consecutive note durations. Each R value is thus derived from a series of individual note durations that band together to form a durational contour. The averaging leads to the possibility that two different instances of rubato may share very similar R values while differing in their internal distribution of time intervals – differing, indeed, in their rhythmic 'feel.' Conversely, two different R values that point to contrasting rubato magnitudes fail to reveal any potential structural resemblance between their internal note-duration profiles. In short, the value of R does not reflect the shape of the contour formed by note durations. While R provides a useful measure of rubato depth for any given moment in the performance, a different – and complementary – analysis is needed to ascertain how the temporal fluctuation is shaped by a given sequence of note durations.

Drawing on a corpus of 28 recordings of Schumann's Träumerei, Repp (1992a) showed that the same motivic fragment or 'melodic gesture' can be played in different ways in different performances. Where one pianist might time a particular melodic gesture as though it were smoothly U-shaped, another might convey a more jagged impression.[17] As in Repp's study, we apply a statistical technique known as principal components analysis (PCA) to identify contrasting timing profiles.[18] PCA is a standard technique for detecting significant deviations from the mean (see Appendix B). Each data point is assumed to consist of a sequence of numerical values (a vector) such as, for instance, a contour of note durations constructed for each version of the Prelude around each rubato barline.

Figure 8 illustrates timing contours from three performances as follows. Note durations are shown for the last four semiquavers of one bar and the first four semiquavers of the next bar. (Note that these graphs plot actual durations – in seconds – for individual notes, in contrast to the earlier graphs showing context-dependent R calculations of rubato depth.) The empty dots in Tilney's graph denote the durations of the eight semiquavers that surround the barline at 1-2: the last four notes (13-16) of the previous bar and the first four notes (1-4) of the new one. There is an obvious peak at note 16 followed by a more or less gradual resumption of speed. The next barline, plotted in filled dots, exhibits a very similar contour. Levin suppresses note 16 and channels his rubato into a sharply delineated agogic stress on the downbeat. The bottom graph illustrates Van Asperen's consistent undulations. In all graphs, the sizeable sweeps of ~150 milliseconds along the vertical axis help to place the timing shapes in perceptible territory. These profiles appear to be staples of each performer's rhythmic identity.

[17] See also Widmer (2002) for an analysis of melodic timing profiles in a corpus of Mozart piano sonatas.
[18] Principal components are also used in Repp (1998b), but those duration profiles cover much wider spans.

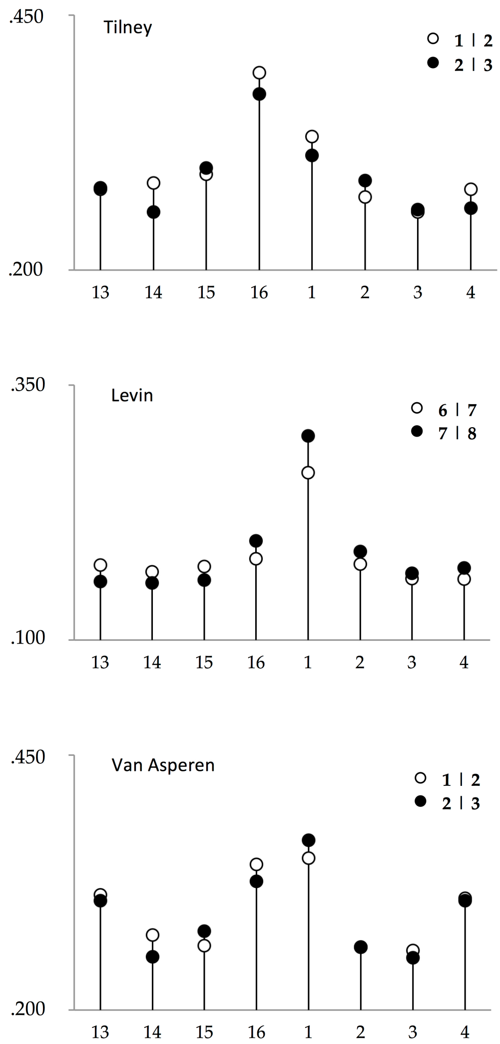

Figure 8. Duration contours spanning eight notes (two beats) in two contiguous bars. Notes 13 to 16 are the last four semiquavers in the bar; note 1 is the downbeat.

We refine the above observations by defining a family of representative timing profiles that will serve as reference templates. The corpus contains 451 rubatos at the barline (with R's threshold set at t = .045), each with a corresponding timing vector containing notes 13 to 4 (the semiquavers in beats 4 and 1). This window of eight notes is smaller than the one used to calculate R, which covers a (weighted) range of 24 notes. The reason for the difference has to do with the effect being portrayed: R requires a wide window to enable a tempo comparison between a region of notes and its two flanking regions, while the timing contour seeks to characterize the rhythmic 'feel' of a local moment.

We first normalise the vectors so that each has a mean duration of 0 and a standard deviation of 1. This retains the shape outlined by the eight durations contained in the vector, irrespective of tempo. Figure 9 shows the cloud of 451 vectors plotted on the two most important principal components (PCs) – that is, those PCs that together explain just above 76% of the variability in the data. The first and second PCs lie on the horizontal and vertical axis, respectively. Every vector in the corpus can be represented as a weighted combination of these two PCs.

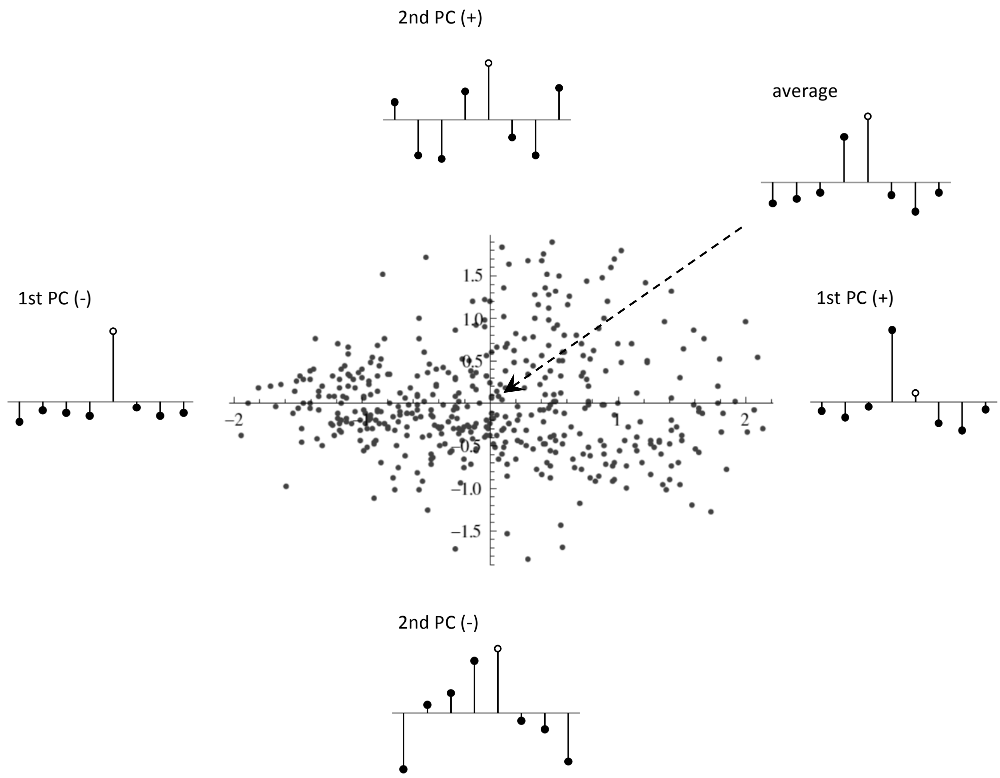

Figure 9. PCA of timing profiles spanning notes 13 to 4 in rubatos at the barline. The first PC (x-axis) is responsible for most of the variance; it reflects the contrast between the durations of the last note in the bar and the first note of the next bar (unfilled circle). The second PC (y-axis) reflects the contrast between the durations of the outermost pairs of notes – whether notes 13 and 4 are both long or both short.

Averaging the 451 vectors gives the central point (henceforth 'average vector') from which the covariances of the PCA are calculated. To obtain representative timing contours, we travel outward along four orthogonal directions from the graph's origin (where the average vector is located) until we reach the far fringes of the cloud. In other words, the reference profiles shown in Figure 9 correspond to opposite extremes along each of the two PC axes. The first PC's principal source of variance is associated with the choice of lengthening either the last or first semiquaver in the bar. (The unfilled dot denotes the downbeat.) The second PC's distinctive feature rests primarily on the relative slowness of the outermost notes: whether the contour outlines (roughly) a W or an inverted V. The former represents a lurchy stop-and-go motion; the latter, an easing into the downbeat followed by a somewhat sudden resumption of tempo. Two PC pairs plus the average vector yield a total of five reference prototypes. In all cases, it is the manner by which the downbeat is prepared and departed that defines the difference between the contours.

RUBATO VS. GROOVE

Some performers appear to gravitate consistently toward specific regions of the cloud. Performers who use frequent, consistent rubatos can be identified by what might be termed their 'microrhythmic signatures.' This observation is consistent with Justin London's 'many meters hypothesis,' which states that

> …we acquire our metrical listening habits by listening to real-world, human performances of music …. [T]heir timing patterns are stable, involving expressive nuances that … may be highly individuated – not just among substyles of a music … but also in the idiosyncratic rhythmic behaviors of particular musicians (London, 2012, p. 8).

The five harpsichordists represented in Figure 10 can be seen to employ a recurring temporal pattern that becomes recognisable and predictable.[19]

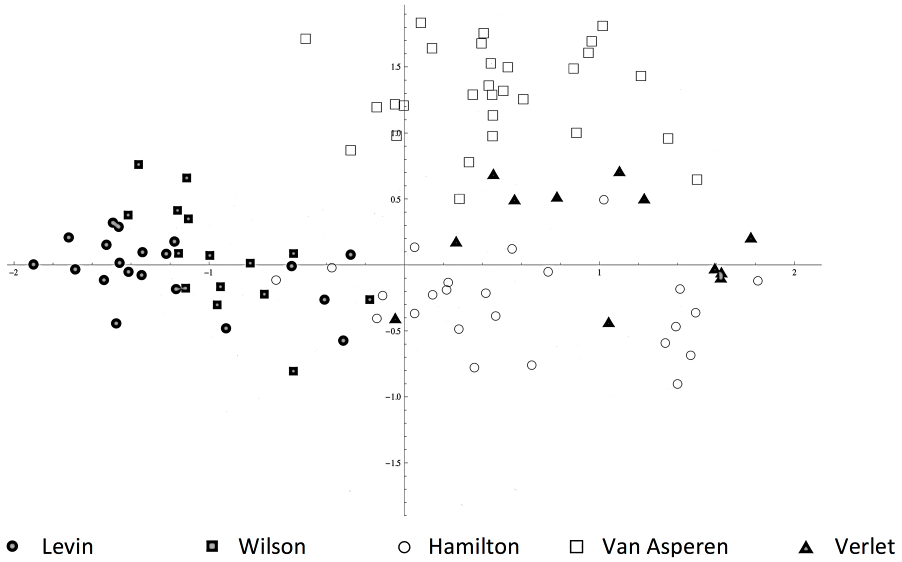

Figure 10. Individual approaches to timing represented on the PC cloud.

This kind of pattern consistency can be thought of as a 'groove': "a perception of a cycle in motion, a form or organizing pattern being revealed, a recurrent clustering of elements through time. Such consistent, coherent formal features … are uniquely recognizable" (Feld, 1988, p. 74). In other words, a groove is a recognisably consistent cyclical clustering. (Note that the groove considered in this article is microtemporally non-isochronous, unlike isochronous, get-up-and-dance types of groove.) Such grooves can be generated by as little as the repetition of a single elongated note, as in Levin's performance, or as much as a two-beat span, as in Van Asperen's performance.

A groove may also be longer than the two-beat span covered by our PCA. (We chose a window of two beats as a compromise between capturing a sufficiently large sample of durations and excluding durations that may not play a role in determining the contour's salient features.) Povel (1977) and Cook (1987) have already pointed out inter-performer differences in different renditions of the Prelude. For instance, Cook discusses at length the ways in which Walcha's and Gould's mean bar-wide (i.e., from note 1 to note 16) contours differ from each other, not only in terms of the way they are represented visually on the graph but also their "just about audible" temporal properties such as "kick," "emphasis," and "grouping by threes" (Cook, 1987, p. 262). The two performers' mean bar-wide contours are shown alongside Egarr's in Figure 11.

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[19] Koren and Gingras (2014) studied musicians' and non-musicians' ability to recognize performances by different harpsichordists. Tempo was the main determining factor leading to recognition, although note onset asynchrony was also a contributing factor.

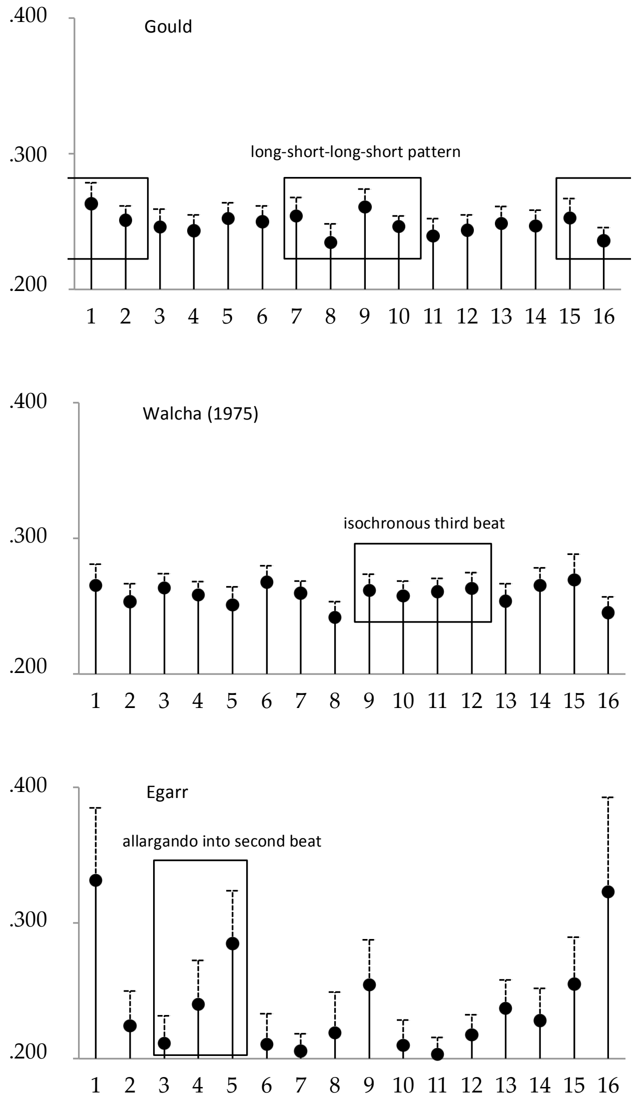

Figure 11. Bar-wide timing profiles reveal features within the bar: a \/\ shape at the half-bar (Gould), strict isochrony through the third beat (Walcha), and an allargando into the second beat (Egarr). Error bars give standard deviations.

Considering all 16 subdivisions allows us to observe timing features that may lie beyond the barline. For Egarr, we note the pronounced allargando into the fifth semiquaver; Gould's half-bar echoes the four-note \/\ shape enclosing the barline; Walcha's third beat is particularly isochronous. But more importantly, and given the three performances' similar tempo of ~60 bpm, it is clear that the y-axis values in Walcha's and Gould's graphs have a much smaller range than in Egarr's. His fluctuations appear vertiginous and are likely, therefore, to be easily perceived; theirs appear inconspicuous to the point where they must be perceived at the subliminal level, if at all.

How does the concept of individual grooves fit with the notion that performers often use timing to highlight phrase structure? To help disentangle groove from rubato, we can think of rubato as a general state of microtemporal non-isochrony, but without the strict groove-like conditions of recognisability, consistency, cyclicality, and clustering. The graph of Egarr's performance in Figure 4 shows higher R values at important structural points – a hallmark of rubato. We would therefore argue that while rubato is undoubtedly present in his performance, it co-exists with a different rhythmic strategy, that of a particular groove. The two are deeply intertwined here, but need not always be.

Van Asperen, for instance, plays with a groove and without rubato. Figure 12 provides a visualization of Van Asperen's performance in which duration is represented as elevation from an aerial perspective; time is shown to unfold as in Figure 7 from left to right. The consistency of his timing is visible as a pattern of recurring wave-like shapes. There is no emphasis to be seen at any of the hypermetric downbeats. As far as timing goes, Van Asperen seems to pay no heed to structural considerations. If there is rubato here, it is submerged below the tide of groove.

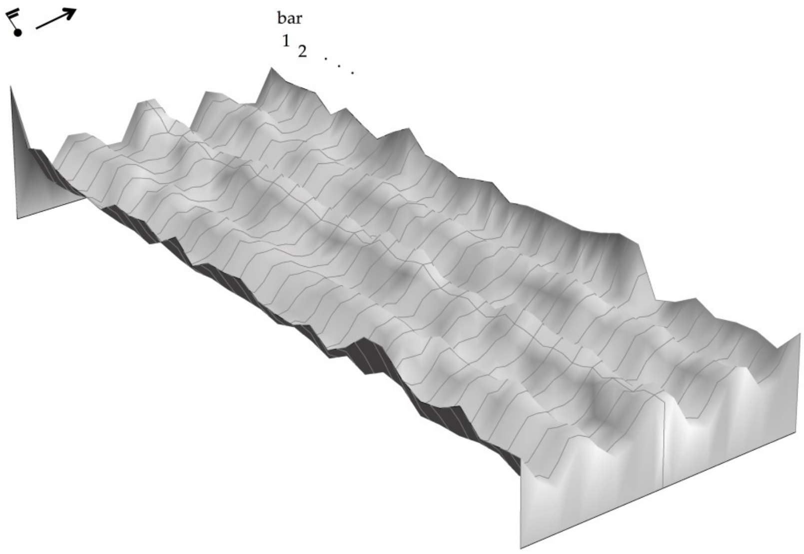

Figure 12. Aerial view of Van Asperen's timing, bars 1 to 31. Time unfolds as in Fig. 7: the piece begins on the upper left, with semiquavers running diagonally from left to right and bars proceeding lengthwise from upper left to lower right. The lack of salient peaks, combined with the uniformity of the folds, points to a groove-based performance devoid of rubato.

CONCLUSION

Rubato is typically depicted as a brief letting up and subsequent resumption in the flow of musical time. This article expands on that understanding by showing that rubato encompasses rhythmic properties that are best examined with different analytical strategies. The Prelude's uniform surface allowed us to examine these properties without fear of interference from foreground motives or rhythms that might elicit sporadic timing behaviours of their own.

We began by showing that performances of the Prelude's repetitive figure are subject to varying degrees of temporal fluctuation, most frequently around the barline. The magnitude of the fluctuation was quantified with an index R representing rubato depth. By compressing a series of (often jittery) interonset intervals into a single value, R provided a quantifiable snapshot of a musical event. This allowed us to construct meta-snapshots of entire performances that could be compared to each other as well as juxtaposed against the

Prelude's harmonic roadmap. We addressed the perceptual relevance of R by setting a minimum threshold value obtained from a listening experiment with expert musicians.

Beyond the general tendency to employ rubato at structural boundaries (an unsurprising finding given the cited literature on performance timing), and beyond the observation that these boundaries may be out of sync with alternative grouping structures realised by the performer (Ohriner, 2012), our analysis showed that rubato is used to enhance linear motion in the Prelude's outer voices. This was noted in stepwise downward resolutions of dissonant tones in the bass line (see Fig. 5) and in the registral peak of the ascending soprano line over the dominant pedal (see Fig. 6). In the latter case, note lengthenings occur around the bar's (weak) second beat – the onset of the top voice – rather than around the downbeat, amounting to an appreciable shift from the rhythmic patterning of the preceding bars (see Fig. 7).

Using principal components analysis, we then shifted our focus to examine note-duration profiles that were directly derived from the (eight-dimensional) universe of 451 rubato vectors. Alternatively, we could have put forth pre-determined plots – the parabola makes a strong showing in the microtiming literature – and sought best-fitting scores between them and the vectors in the corpus. We opted for the present method because it avoids preconceived notions about what rubato ought to be. Indeed, we found distinctly contrasting timing configurations and argued that, when deployed consistently, these take on a groove-like quality that is not fully captured by the implications behind the term 'rubato.' This distinction underscores the need for the further development of precise terminology in timing analysis.

In light of this, it is helpful to think of a performer's timing approach to a given performance as lying on a self-consistency continuum. On one end of this continuum, we find groove-like performances that use a recurring temporal template; on the other end lie performances with variable timing routines. Thus, two Prelude renditions that differ dramatically in their amount of rubato, such as Gould's and Egarr's, can nonetheless be considered equivalently individualised from a microtemporal perspective. Admittedly, assessing self-consistency in this context is computationally problematic. A mostly rubato-less performance may be temporally consistent except at select moments of pronounced and inconsistent rubato; this amounts to a composite of two timing strategies, of which only one is self-consistent. And, as discussed above, some templates require a wide swath while others make do with a single note, a matter not easily translated to the language of quantitative analysis. Nonetheless, a useful conceptual outcome of this mapping is that it begins to disentangle groove-like consistency from rubato depth – two important yet separable features of timing.

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Appendix A. The 48 performances analysed in this study.

Performer	Instrument	Year	Duration
Aldwell, Edward *	piano	1992	2:45
Ashkenazi, Vladimir	piano	2005	2:41
Backhaus, Wilhelm	piano	1937	2:03
Barenboim, Daniel	piano	2003	1:41
Busoni, Ferruccio	piano	1922	1:45
Cohen, Harriet	piano	1928	1:44
Egarr, Richard	harpsichord	2003	2:30
Feinberg, Samuil	piano	1958	2:31
Fellner, Till	piano	2002	2:04
Feltsman, Vladimir *	piano	1992	2:23
Fischer, Edwin	piano	1933	1:24
Gieseking, Walter	piano	1950	1:28
Gilbert, Kenneth *	harpsichord	1990	2:09
Gould, Glenn ‡	piano	1962	2:23
Gulda, Friedrich *	piano	1972	2:08
Hamilton, Malcolm	harpsichord	1965	2:42
Hantai, Pierre	harpsichord	2001	2:07
Hewitt, Angela	piano	1998	2:09
Hewitt, Angela *	piano	2009	2:21
Horszowski, Mieczyslaw	piano	2003	1:48
Jarrett, Keith *	piano	1987	1:54
Kirkpatrick, Ralph	harpsichord	1963	1:47
Kirkpatrick, Ralph	clavichord	1959	1:52
Koopman, Ton	harpsichord	1982	2:51
Landowska, Wanda ‡ *	harpsichord	1949	2:27
Leonhardt, Gustav †	clavichord	1989	1:45
Levin, Robert *	harpsichord	2000	1:44
Loesser, Arthur	piano	1961	1:30
Martins, João Carlos	piano	1981	4:03
Moroney, Davitt	harpsichord	1988	1:54
Nikolayeva, Tatyana	piano	1972	2:15
Pinnock, Trevor	harpsichord	1991	2:16
Pollini, Maurizio *	piano	2008	1:52
Rangell, Andrew	piano	2007	1:50

Richter, Sviatoslav ‡	piano	1970	1:55
Riefling, Robert	piano	1985	1:54
Ross, Scott	harpsichord	1980	2:03
Samuel, Harold	piano	1926	1:47
Schiff, Andras ‡ *	piano	1984	1:54
Tilney, Colin	clavichord	1988	2:54
Tureck, Rosalyn *	piano	1953	2:59
Tureck, Rosalyn	piano	1990	2:36
Van Asperen, Bob	harpsichord	1989	2:49
Verlet, Blandine *	harpsichord	1993	1:58
Walcha, Helmut †	harpsichord	1960	2:25
Walcha, Helmut ‡	harpsichord	1975	2:33
Wilson, Glen	harpsichord	1989	1:57
Woodward, Roger	piano	2009	1:59

† Also analysed by Povel (1977)
‡ Also analysed by Cook (1987)
* Used as stimulus in the present experiment

Appendix B

1. Calculation of the rubato index

The rubato index R assigned to the nth note of each version of the Prelude is a weighted average of the measured durations from the (n–11)th note to the (n+12)th note. Mathematically, it can be written as

$$R = w_{n-11}\, d_{n-11} + w_{n-10}\, d_{n-10} + w_{n-9}\, d_{n-9} + \ldots + w_n\, d_n + \ldots + w_{n+11}\, d_{n+11} + w_{n+12}\, d_{n+12}$$

where d_i ($i = n$-11, … , n+12) is the duration of the ith note, measured in seconds, and w_i is the corresponding weight.

Weights were extracted from a Mexican-hat algebraic function of the form $f(x) = A\,(1 - a\,x^2)/(1 + b\,x^4)$ for integer values of the variable x from x = -12 to -1 and from x = 1 to 12. The coefficients a = 0.202 and b = 0.047 were chosen in such a way that weights are positive for notes n-1 to n+2 and negative for the others, and that the sum of all weights from note n-11 to note n+12 equals zero. The arbitrary factor A = 1.149 fixes the scale of rubato values. The numerical values of weights are

$w_{n-11} = w_{n+12} = -0.033$ $\quad$ $w_{n-5} = w_{n+6} = -0.117$

$w_{n-10} = w_{n+11} = -0.039$ $\quad$ $w_{n-4} = w_{n+5} = -0.153$

$w_{n-9} = w_{n+10} = -0.047$ $\quad$ $w_{n-3} = w_{n+4} = -0.197$

$w_{n-8} = w_{n+9} = -0.057$ $\quad$ $w_{n-2} = w_{n+3} = -0.196$

$w_{n-7} = w_{n+8} = -0.071$ $\quad$ $w_{n-1} = w_{n+2} = 0.125$

$w_{n-6} = w_{n+7} = -0.090$ $\quad$ $w_n = w_{n+1} = 0.875$

The Mexican-hat profile of weights defines the rubato index R as a kind of smoothed second derivative (or discrete Laplacian operator) of note duration as a function of time, with its sign changed. Since the total sum of weights vanishes, and because of their left-right symmetry, R is identically equal to zero in zones where the durations of notes increase or decrease linearly with time. More generally, R is close to zero where durations vary monotonically along the sequence of notes. On the other hand, R is sensibly positive or negative where durations attain a maximum or a minimum, respectively. Large positive values of R are thus obtained around relatively longer notes.

2. Principal component analysis (PCA)

Mathematically, PCA identifies – by means of a rotation of data space around the data average – an orthogonal coordinate system such that the projections of the data onto its axes become uncorrelated to each other. This is equivalent to detecting the principal axes of the ellipsoid that best approximates the cloud of data points around their average, which coincide with the directions of maximum relative variation of the data. The first principal component (PC) is the axis along which the data exhibit their largest dispersion (in both directions). The second PC is the direction of maximum dispersion if the projection of each data point along the first PC is disregarded, and so on. Most advanced software packages for algebraic and data manipulation include predefined functions with automated implementations of PCA. In the present work, PCA of rubato profiles was performed using Wolfram Mathematica®.

Appendix C. Full Score