Dept. for Speech, Music and Hearing

Quarterly Progress and Status Report

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Kronman, U. and Sundberg, J.

journal: STL-QPSR
volume: 25
number: 2-3
year: 1984
pages: 126-141

B. IS THE MUSICAL RETARD AN ALLUSION TO PHYSICAL MOTION?

Ulf Kronman* and J. Sundberg

Abstract

Is the musical retard an allusion to a physical deceleration? This question is investigated by the method of model-to-measurement matching. The model is derived from considerations of moto-rhythmic motion, i.e., motoric motion giving rise to impulses whose density in time has a direct relationship to the velocity of movement. The measurements originate from an investigation of 24 final retards in performances of motor music, i.e., music dominated by long sequences of short and equal note values. The model predicts tempo changes falling within one standard deviation of the average of these retards except for the last data point. However, an almost perfect match between model and measurements is gained if the endpoint of the retard is assumed to appear on the average 10% beyond the onset of the final chord. This implicates the beat tempo at the onset of the final chord to be 30% of the initial beat tempo.

Introduction

In Western classical music only part of the sound events are included in the written music score; a good deal is added by the performer. One of the most commonly added concepts is the final retard often ending a piece of music. By means of the sign "Rit." the composer usually leaves the execution of the retard free for the musicians or the conductor to interpret. Even though the exact design of the retard appears free, there seems to be some concensus among competent listeners as to what is an acceptable final retard. A linear decrease of tempo throughout the retard, for instance, is judged as "poor" in most cases (Sundberg & Verrillo, 1980; henceforth S&V). Different performed final retards appear to have something in common that makes them meaningful signs of the ending of the piece. The question we shall ask here is on what ground this common denominator is based, or, more specifically, what references might be involved.

Another fact, that is perhaps less known, is that a skilled performer has to deviate from the exact notated durations throughout the piece in order to create a musically convincing rhythm (Bengtsson & Gabrielsson, 1983). One may assume that such deviations from the notated durations provide some sort of information to the listener, e.g., by means of associations (S&V; Sundberg, Frydén, & Askenfelt, 1983; Bengtsson & Gabrielsson, 1983).

It may further be suggested that music based on a metrically monotonous ground, which will henceforth be referred to as "moto-rhythmic" music, carries associations with a listerner's experience of physical motion; the sequence of impulses we perceieve when we walk or run is rather similar to the regular sequence of tones in moto-rhythmic music.

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* thesis student in music acoustics.

If the music reminds the listener of physical motion, it would be natural to insert a retard, because we know from experience that a physical motion is preferably slowed down before it stops.

As a musical retard may be described in terms of variations in tempo and rhythm, one of the most obvious associations to consider with regard to retards would be the motional aspect of the tempo-changes. The hypothesis to be investigated here is whether the retard can be seen as an allusion to a physical deceleration. If our experience with physical motion serves as a frame of reference for the interpretation of the performance of a retard, it would be possible to make a model for physical deceleration and match this model to measurements of some typical retards.

The method of investigation will be as follows:

A. Development of motion model.

B. Study of retards.

C. Matching of model to averaged retard measurements.

D. Matching of model to single retards.

Investigation

As the terms "rhythm", "tempo", "metre" and "beat" will be frequently used in this investigation some clarifications may be needed. Some of the following definitions are taken from an article on the origins of tempo behaviour (Brown, 1979) and have been adapted to the present purpose.

Beat - The unit of musical movement. The beat is usually given by the denominator of the time signature and would probably be the rate of foot tapping during playing or listening.

Metre - is stated in the numerator of the time signature and marks the grouping of beats. The metre indicates how many beats it takes to complete a measure (e.g., 2/2, 3/4, 4/4 or 6/8).

Tempo - is defined as the inverse of the duration between the onsets of single tones. Observe that beat and tempo are not the same; - a melody with a slow beat can still be played in a rapid tempo (e.g., if the metre is 2/2 and the melody is made up of semiquavers) and vice versa.

Instant tempo - is the inverse of a single onset-onset time between two succeeding tones.

Mean tempo - is the inverse of the average duration measured over many tones.

Beat tempo - is the inverse of the duration of the beat. The term emphasizes the beat as having an instant rate which can be measured and calculated in the same way as with single tones.

Rhythm - is a perceived characteristic of a group of acoustic signals. In practical performance the rhythm can be seen as the proportional relationship and tension between the single tone durations and the regular underlying beat and its metre.

A. Model for deceleration of physical motion

We will limit our study to the types of motion which can be classified as moto-rhythmic, i.e., "rhythmic" in a motoric sense. A moto-rhythmic motion is any physical motion which generates impulses whose density in time has a direct relationship to the velocity of the motoric movement. Examples of such moto-rhythmic motion is, for instance, human or animal walking or running, train running or other motions where the step-length, i.e., the distance moved for each impulse, can be considered to be independent of the velocity and, thus, introduced as a constant (c).

For such moto-rhythmic motion the relation between velocity (v) and pulsation rate (T) can be written:

$$v = c * T \tag{1}$$

(v = velocity (m/sec), T = tempo (impulses/sec), and c = distance-per-impulse-constant (m/impulse)).

In analogy with this, the distance (x) as a function of number of impulses (n) will be:

$$x = c * n \tag{2}$$

(x = distance (m) and n = number of pulses)

If a motion with the initial velocity v_o is to come to a complete halt (v = 0) in the distance S by a constant (negative) acceleration (a) the acceleration is given by:

$$2aS = v^2 - v_o^2 \tag{3}$$

(v = 0)

$$a = -v_o^2/2S \tag{4}$$

The instant velocity (v) as a function of distance (x) from initiation of deceleration is given by the same equation:

$$2ax = v^2 - v_o^2 \qquad v = \sqrt{v_o^2 + 2ax} \tag{5}$$

Combining (4) and (5) gives:

$$v = \sqrt{v_o^2 - v_o^2 * 2x/2S} \tag{6}$$

$$v = v_o \sqrt{1 - x/S} \tag{7}$$

In order to get an expression of the instant pulse-tempo (T) as a function of the preretard tempo (T_o) and the distance (x) from beginning of deceleration we combinine (1) and (7):

$$T = T_o \sqrt{1 - x/S} \tag{8}$$

To complete the analogy with music, we substitute the distances x and S with the number of pulses according to relation (2), thus getting:

$$T = T_0 \sqrt{1 - n/N} \qquad (N = S/c) \tag{9}$$

where n is the number of pulses since the beginning of retardation and N is the total number of pulses in the entire deceleration process.

Finally, expressing the tempo as a function of number of pulses (P) left to the final pulse, where the velocity is zero, we get the "generalized retardation function":

$$T = T_0 \sqrt{P/N} \qquad (P = N - n) \tag{10}$$

The result can be seen in Fig. 1. It constitutes the simple model for retardation of physical motion which henceforth will be referred to as a "retardation curve" in contrast with the measured musical "retard curves" to be presented further down.

Given this model of rhythmic deceleration we need to know more of the retards we are going to compare it with.

B. Anatomy of the musical retard

In an attempt to describe an aspect of musical timing, Sundberg and Verrillo analyzed 24 recorded final retards in motor music, i.e., music dominated by long sequences of short and equal note values. Most of the music was composed by J.S. Bach (whose preludes and fugues are good examples of such motor music) and played mainly on the harpsicord.

In the analysis of the retards the instant tempo (T) was defined as the inverse of tone duration. The length of the shortest note value was chosen as duration unit. The durations of the shortest note values were measured over a longer period of time to provide information on the preretard mean tempo (T_0) in which the piece was played.

The retard length was defined as the number of shortest note values from the beginning of the final sequence, in which all notes were played slower than the preretard mean tempo, to the onset of the final chord.

The inverse of the tone durations (representing the instant tempo) was plotted according to their distance from the final chord, measured in number of shortest note values. An example of such a "retard curve" obtained by this procedure is shown in Fig. 2, together with the notation of the corresponding last three measures.

The retard curves were normalized with respect to retard length (N) and preretard mean tempo (T_0). An "average retard" was calculated from the 24 recordings by linear interpolation of tempo and computation of averages and standard deviations at each tenth of the retard. The result can be seen in Fig. 3.

The retard curves were found to exhibit the following characteristics:

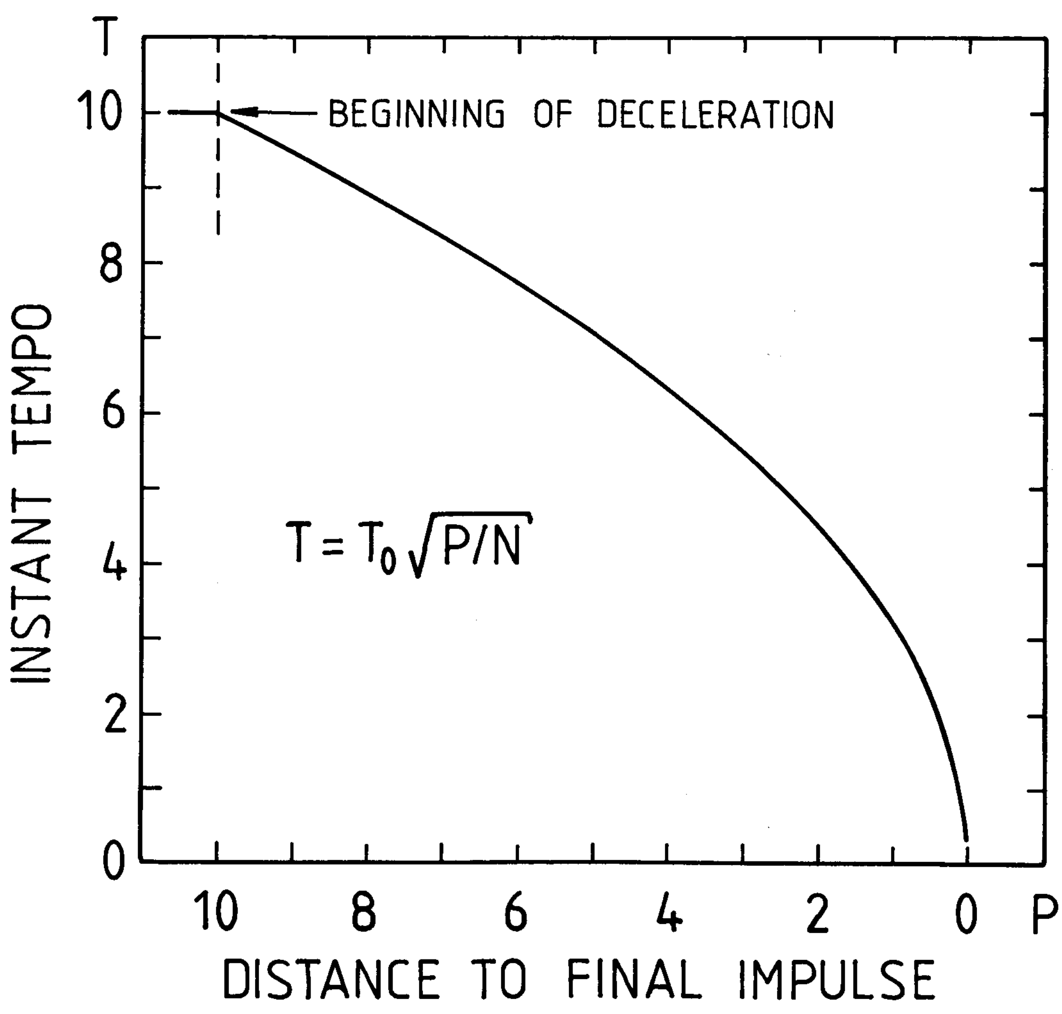

Fig. 1. Retardation curve with preretard tempo T_0 = 10 and retardation length N = 10.

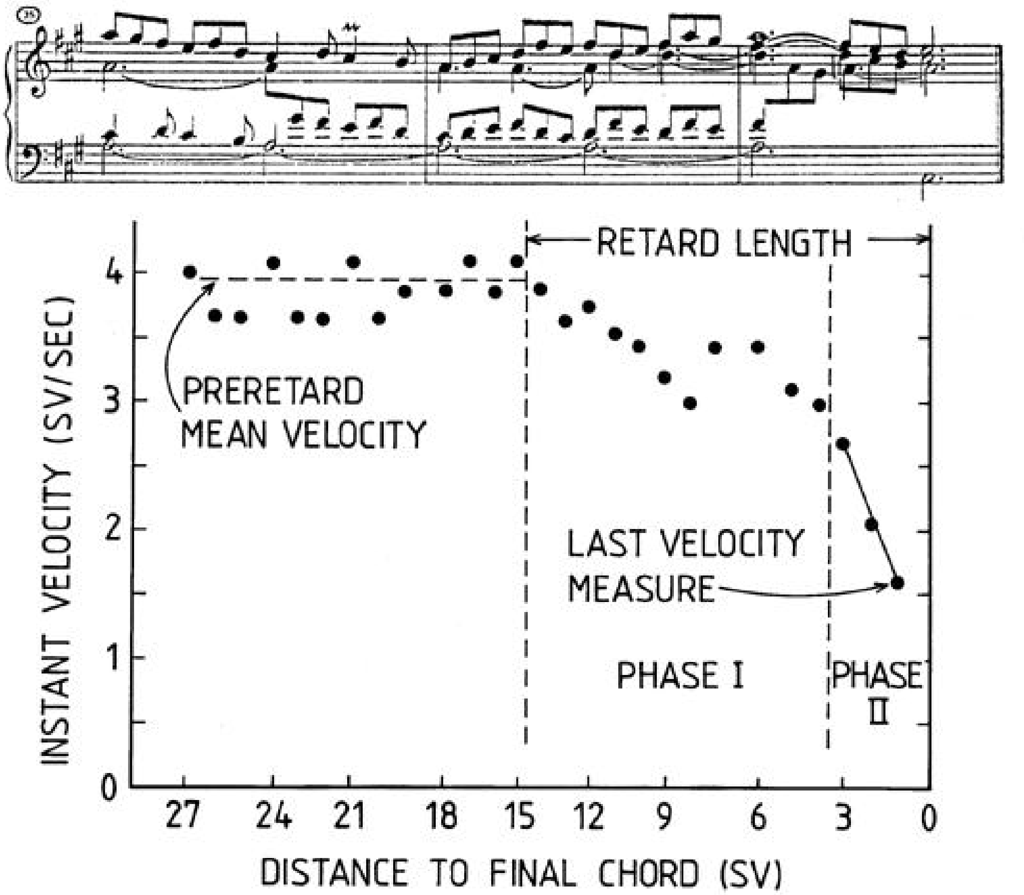

Fig. 2. Example of retard curve (lower graph) describing the timing of the final portion of a piece, the notation of which is shown above. (From Sundberg & Verrillo, 1980).

* Retard length, measured in beats, was related to the length of the final cadence, i.e., the last presentation of the dominant-tonic chord sequence.

* The retards could be divided into two parts; phase I which was characterized by a fairly great irregularity, and phase II which could be described as a systematic linear decrease of tempo. The length of phase II showed to be equal to the length of the last motive, defined as the shortest sequence of notes that constitute a melodic gestalt.

Leaving the S&V study and the description of their observations, we now return to our main task, namely to fit our own model of physical deceleration (Eq. 10) to the measured retards.

C. Matching of model to average retard measurements

If we normalize our retardation curve with respect to preretard mean tempo (T_O) and retardation length (N), we get the dimensionless function:

$$T_N = \sqrt{P_N} \tag{11}$$
$$(T_N = T/T_O,\ P_N = P/N,\ 1 > P > 0)$$

This function is easy to match to the normalized average retard curve mentioned above. The value of P_N will here represent the fraction left in the retardation process.

In Fig. 4 the normalized retardation curve is compared with the normalized average retard curve in Fig. 3. The bars, representing a ± 1 standard deviation, cover the marked points of 16 out of the 24 analyzed retards. Note that the square root curve falls within the bars except for the point corresponding to the last tenth of the retard ($P = 0.1$). This suggests that the theoretical curve can be regarded as a good approximation of all measured tones except those located in the last tenth before the final chord of the retard.

The very last point ($P_N = 0$) represents the onset of the final chord. However, there is no observed value associated with it in the S&V study. The reason is that this point was regarded as the endpoint of the retard, so that no tempo value could be determined at the onset of the final chord. This means that the empirical "curve" has no values for the fraction where $P_N < 0.1$, and, hence, matching is impossible in this region.

These facts direct our attention to some ambiguity regarding the endpoint of a retard that has to be further elucidated before we can proceed.

Where is the endpoint of a retard? For practical reasons, the onset of the final chord was chosen as the endpoint in the S&V investigation (it is often difficult to determine the duration of the final chord, at least in performances on harpsichord). However, neither the instant tempo, i.e., the inverse of the duration of a single tone, nor the beat tempo can be assumed to be zero at this point; how would a harpsichord player know when to take the hands off the keyboard, if he had not a feel of (beat) tempo during the final chord?

Against this background it seems practical to reformulate the question above: What is the beat tempo at the onset of the final chord? This tempo can be estimated using the S&V article: the last part of the retards was found to exhibit a linear decrease in tempo and could, therefore, be approximated as a straight line in all except four of the 24 retards. Its length was defined as the part of the curve where the data points fell close to this line; in half of the cases it included just three data points, but in some cases up to seven points.

If we temporarily accept a straight line to approximate the last retard points of the normalized average retard curve, the function $T_N = P_N + 0.30$ offers a good linear approximation of the three last points. Extrapolation of this line to the point where $P_N = 0$ suggests the tempo to be 0.30 at the onset of the final chord, or, in other words, 30% of the preretard average value. This extrapolated function can be seen in Fig. 5.

According to the reasoning above, a curve describing the decrease of beat tempo must be extended beyond the onset of the final chord and assume the value of .3 at the onset of this chord. If we modify the theoretical retardation curve accordingly, the retardation time will increase by 10 %. This modified retardation curve offers a good approximation of the retard curve. Not only does the theoretical curve fall within the one-standard-deviation bar at the last point, but also do all predicted values fall within ±0.5 standard deviation of the corresponding mean values.

The function suitable to describe this extended retardation is given by:

$$T_N = \sqrt{(P_N + e)/(1 + e)} \qquad (12)$$

(e = extension = 0.1)

and can be studied in Fig. 5 together with the mean values and the standard deviation bars from Fig. 4.

As the lengthening by 10% refers to an average curve, it must be regarded as a mean value itself; this means that the actual lengthening in the individual case could range from 4 to 14%, and still lie within the limits of one standard deviation. This, of course, would depend on factors such as individual mean tempo, retard length, construction of specific retard, etc.

Before continuing further, it is appropriate to consider the question what the "average retard" in fact represents. One may argue that in reality there is no such thing as an average retard; presumably every musically acceptable retard has to be individually designed taking into consideration musical context, instrument properties, room acoustics and so on. An averaging process obviously disregards such individual con-

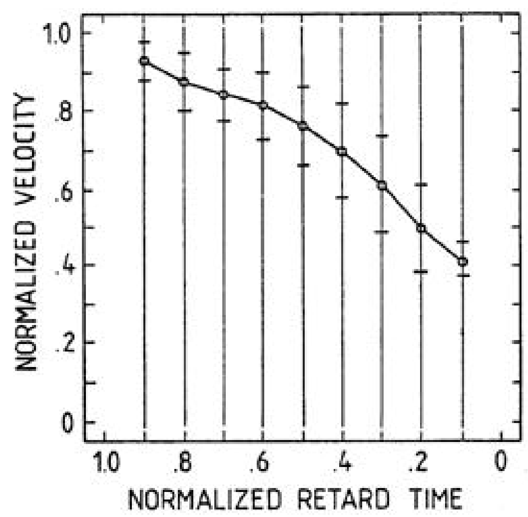

Fig. 3. Averages of the normalized tempo in the 24 retards studied. The abscissa represents the normalized duration of the retard. The bars show ± standard deviation. (From Sundberg & Verrillo, 1980).

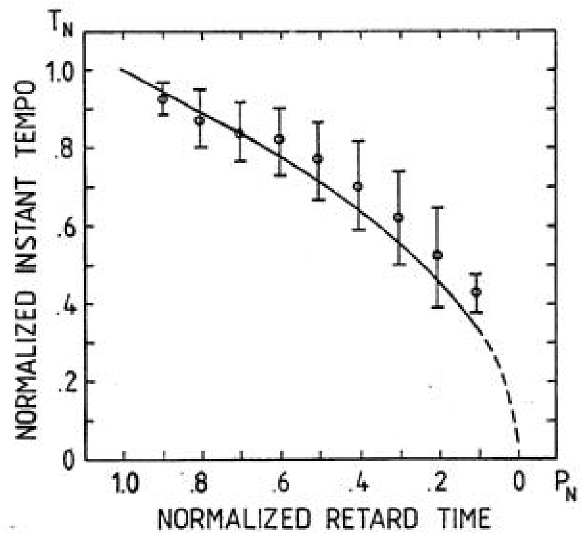

Fig. 4. Comparison of normalized average retard (open circles) and normalized retardation curves (crosses). Bars as in Fig. 3.

siderations. This suggests that the average curve represents the basic retard idea as manifested in the decrease of beat-rate.

We may hypothesize that this beat-rate-skeleton is a sort of development line around which the variations in duration take place. In case of non-retard, this beat rate would be represented by a straight horizontal line, as indicated in the preretard part of Fig. 2. According to this hypothesis, it would be possible to match single retards to the modified retardation curve, thus regarding this beat rate curve as a development line, to which duration corrections are added for expressive purposes. In the next section single retards from the S&V investigation will be examined in this way.

D. Matching of model to single retards

Studying some individual retards, such as in Fig. 2, it appears obvious that a model based on a simple square-root decrease of tempo must be insufficient to explain these rather irregular patterns. There must be a number of musical and technical factors that may influence the performance, and these factors have to be included as parameters in the design of each individual retard.

If we examine the retard curve in Fig. 2, taking into account also the musical notation, we observe that the dotted contour has a "hump" right below the last bar line. The notation tells us that, here, the top voice lands on a chord which allows the second voice to "catch up" with the top voice before the end of the piece. It seems that this retard is in fact made up of three parts: the first part is characterized by a small decrease in tempo and includes points 14-8 in Fig. 6; the second part, comprising points 7-4, begins in a somewhat higher tempo; the third and final part is characterized by a strong retard and corresponds to Phase II in the S&V description.

This specific retard can actually be accounted for using two retardation curve segments, particularly if the retard is lengthened by 10%, as discussed above. This yields a very good agreement, as can be seen in Fig. 6, which also gives the values used for computing this retardation curve.

Looking more closely at Fig. 6, one notices that few of the tempo marks fall exactly on the retardation line. This discrepancy would, at least in part, be due to the fact that the dots refer to the tempo derived from single tones, while the model refers to a beat tempo. To proceed with the assessment of the model, we have to find some adequate relation between beat and single tone tempo.

According to recent investigation of musical performance the duration of the individual tones is influenced by a number of factors (Sundberg, Frydén, & Askenfelt, 1983; Bengtsson & Gabrielsson, 1983). For instance, the last tone or tones in a phrase are often lengthened, as are also target tones in melodic leaps and the first tone after a chord change.

The rules for such expressive deviations proposed in these articles all refer to non-retard performances and, therefore, they may not be

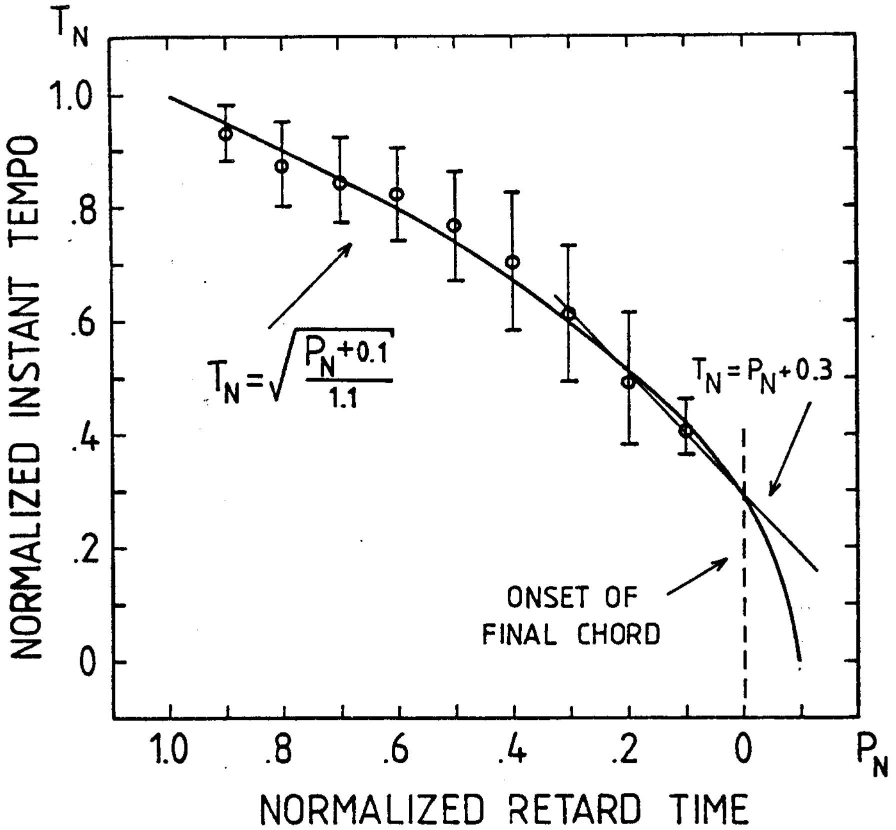

Fig. 5. Retardation curve, extended by 10%, compared with average retard values (open circles), the last three points of which have been approximated also by a straight line; bars as in Fig. 3.

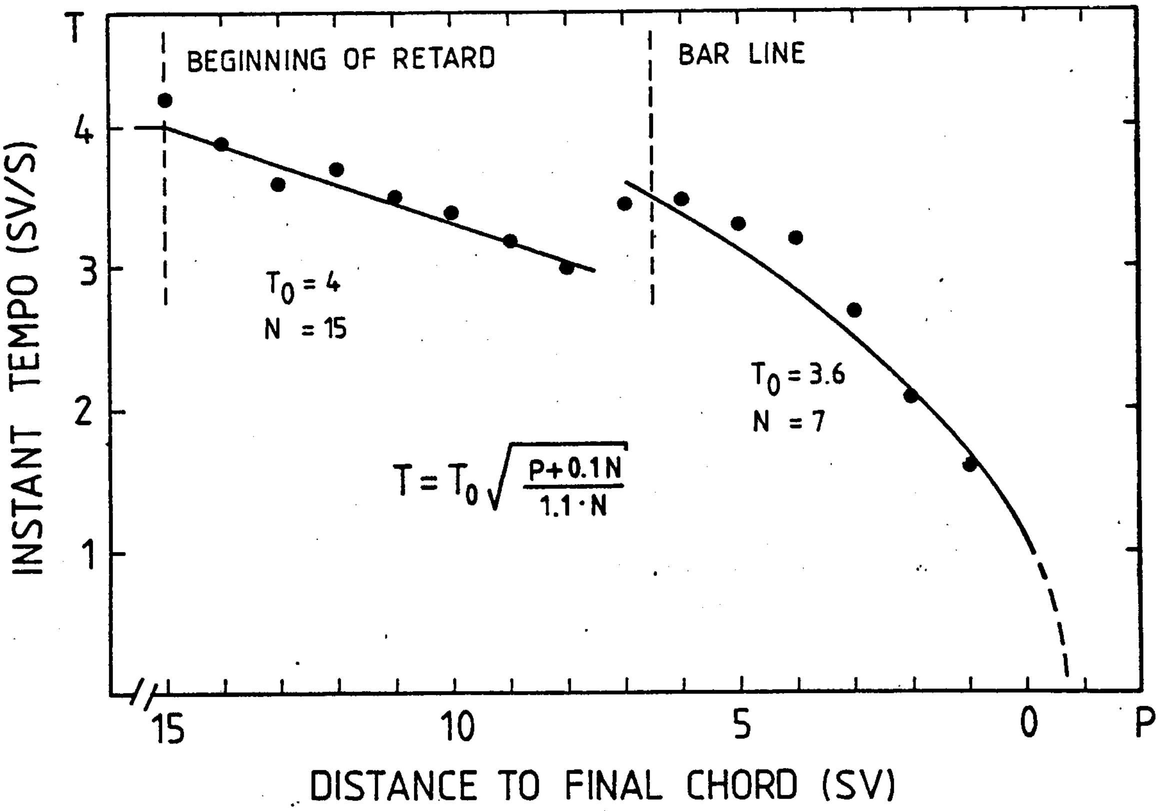

Fig. 6. Performed retard compared with 10% prolonged retardation curve divided into two parts according to the values given in the figure.

applicable to a retard situation. In any event, such rules prove the need for introducing some sort of measure of expressive deviation.

A reasonable assumption would be that the performer uses at least as big margins for expressive deviations during the retard as during the pre-retard performance. This means that such margins should be added to the retardation curve, and we would then expect that the measures pertaining to individual retards should lie within these margins, if the retardation is a good model of the retards. To get a realistic estimate of margins applicable to each case, the performance of the last few pre-retard bars was studied and the measured deviations were expressed in relation to the computed mean tempo. The maximum duration deviations were found to be of the order of magnitude of 10-70 msec with an average of about 40 msec; this results in tempo variations from 4 to 27% of the individual average tempo, with 10% as a mean value.

Another factor of importance is the possibility of unintended deviations, or, in other words, playing errors. Presumably, even the most skilled musician makes small unintended variations due to technical and psychological factors but it is difficult to separate intended from unintended deviations because of the present lack of tools for predicting what is intended. However, both unintended and intended variations are evidently included in the value of the preretard expressive deviations mentioned above. A modified model, including appropriate margins for expressive deviations, should explain the main part of the performed durations in single retards.

The method for adapting the beat retard model to single tone retard measurements will thus be:

I. Choose an optional length of the retardation by placing the endpoint 0-25% beyond the onset of the final chord. (Use the divided retard model shown in Fig. 6, if implicated by performance and notation.)

II. Add margins for expressive deviation according to the maximum deviations indicated by prior execution of non-retard performance of the same piece.

An example of such a curve can be seen in Fig. 7.

Applying the above mentioned procedure, the 24 retards in the S&V study were compared with the model including the expressive deviations. The result showed that the model could explain 82% of the performed durations. In two cases the divided retard model was used.

In order to get an idea of the significance of this result, a simpler, alternative model comprising a linear decrease of tempo was also tested, applying the same conditions as for the previous model. An example is shown in the same Fig. 7. The results showed that this straight line model could explain no less than 83% of the measured durations. However, this required that in many cases the postulated endpoint of the retard had to be placed 50-100% beyond the onset of the final chord. This implies that, for example, the final chord of the retard, shown in Fig. 7, should be sounding in 3.9 sec, which seems unrealistic in view of the fast decay of a harpsichord tone. Moreover, a

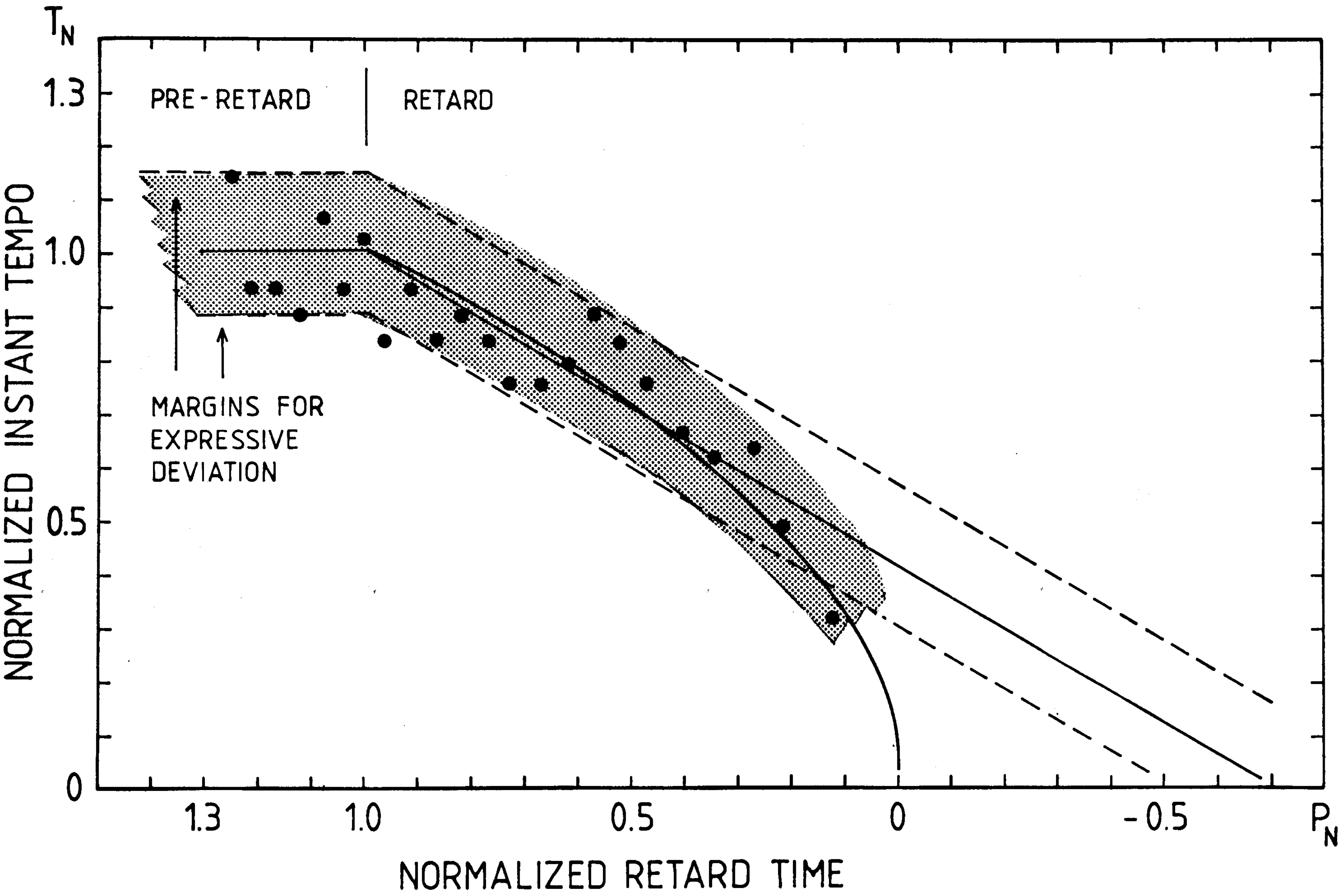

Fig. 7. Predicted curve for beat retard and straight line retard applied to a normalized performed retard. Margins are added for expressive deviation (hatched area and dashed lines, respectively).

straight line retard predicts too high a value for the last tempo measure in 21 cases, and in 8 cases the measured value is even beyond the margins of expressive deviation. For these reasons the straight line model appears to offer a poorer explanation alternative than our square root model.

Summarizing, the study of single retards has shown that the square root decrease of beat rate explains the majority of the performed durations, provided that the location of the point of zero beat can be chosen beyond the onset of the final chord, and provided that sufficient margins are allowed for expressive deviations. The advantage of the square-root model could be further substantiated if knowledge were developed allowing better prediction of the point of zero beat rate in retards.

Discussion

The idea that rhythm in general, and the final retard in particular, alludes to physical motion is regarded as self-evident by many musicians. For instance, rhythm and tempo changes are usually announced by references to physical motion, e.g., "faster" and "slower". This suggests that there is some sort of intuitive consensus among performers and listeners in this respect. However, it is often a complicated task to prove the existence of such intuitively known "facts" in a scientific way. Presumably, this is due to the present lack of tools for separating and understanding scientifically the different parts of the musical message.

The retardation model presented here can be seen as a first step towards an explanation of retards, which hopefully will lead to a deeper understanding of the associative bases for rhythmic structure in general.

The present investigation has demonstrated that the measurement of single tone tempo, though useful in describing performances, is inappropriate in a predictive study of retards, as it cannot account for the possibility that a motion is perceived during the final chord. It seems that using the notion of beat tempo instead of single tone tempo solves this problem.

With respect to the study of beat retard, there are two possible ways to solve the problem of the endpoint. One way is to regard the onset of the final chord as the retard endpoint and to make an estimation of the beat tempo at this point (whether zero or not). Another way is to define the retard endpoint as the time when the beat rate reaches zero and then find a procedure to locate this point. With regard to the model sketched here, the last mentioned method seems preferable. In order to locate the point of zero, beat rate listening experiments would probably be helpful. Also, musicians could be asked to mark the beat (e.g., by tapping the foot) while playing.

Once the point of zero beat rate has been identified, further evaluation of our square-root model can be carried out. For instance, alternative retards can be computed for various pieces of music and their musical acceptability can be evaluated in subsequent listening tests. It would probably be worthwhile to compare retards with beats following our square-root model ($T_N = \sqrt{P_N}$), with retards following a linear ($T_N = P_N$) or "circular" decrease ($T_N = \sqrt{1 - (P_N-1)^2}$) of beat rate.

It has been revealed that animals as well as car drivers instinctively use a visual estimation of the remaining time-to-contact for adjusting their deceleration to a safe value, when they try to stop in front of an obstacle or at a given point (Lee, 1976; Lishman, 1981). The retardation force is then highest at the beginning of the retardation, the force subsequently being smoothly decreased. Evidently, such a retardation will be more time consuming than a retardation using a constant retardation force. However, the retardation curve typically produced by car drivers is close to a retardation curve obtained with a constant retardation force. This shows that our assumption of a constant retardation force is not crucial. Therefore, on the basis of the present data, we cannot judge whether a musical retard is an allusion to a practical braking of a vehicle in motion or to some type of a generalized or "ideal" retardation process. Still, the similarity between the retardation and retard curves suggests that the retard serves the purpose of evoking a listener's associations to some type of deceleration to which is added expressive information.

It is possible that the final retard may inform on other interesting aspects of music communication. For instance, the retard length may be associated with the "mass" in motion, and micropauses between adjacent tones may be associated with the "force" applied to this "mass", to mention two examples. The exploration of such speculations is left to future investigations.

Conclusions

1. Data averaged over 24 motor music retards can be matched almost perfectly by the retardation model, provided that the beat rate is used as the measure of the tempo, and that the retardation is prolonged by 10% beyond the onset of the final chord, implying that the beat tempo at this point is 30% of the preretard tempo.

2. This agreement between the retardation curve derived from the model of moto-rhythmic motion and the measured average final retard in motor music suggests that the musical retard can be seen as an allusion to physical deceleration.

3. Over 80% of the tone durations in 24 single retards are in good agreement with the model, provided a) that appropriate margins are allowed for expressive deviation taking into account various aspects of musical context (bar lines, phrases, etc); and b) that the point of zero beat rate can be chosen beyond the onset of the final chord.

4. In order to refine the model for describing musical retards in more detail, more research is needed in defining the actual endpoint of the retard.

Acknowledgments

The authors are indepted to Anders Askenfelt for valuable discussions.

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